

Learning to Use New Tools:
A study of mathematical software use
for the learning of algebra

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of the requirements for the degree of
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DECLARATION

I, Stephen Mark Arnold, the undersigned, hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person nor material which to a substantial extent has been accepted for the award of any other degree or diploma of a university or other institute of higher learning, except where due acknowledgment is made in the text.

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ABSTRACT

This study documents the efforts of a teacher/researcher to learn to use computer algebra software applications as pedagogical tools through systematic self-study, clinical observations of secondary students and collaboration with groups of preservice teachers. The study also involved the ongoing development of a computer-based learning environment which accompanied the research process and served to embody the main results. Complementing action research methods with grounded theory analysis, the study describes and explains the ways in which individuals (six secondary students and two groups of six preservice teachers) used available software tools for algebra learning. The subsequent grounded theory situates tool use within contexts of mathematical and pedagogical thinking on the part of the user. Effective use of available software tools was also found to be conditional upon characteristics of both the software and the learning environment.

Analysis of pedagogical beliefs of both students and preservice teachers revealed a consistent culture of mathematics learning which devalued external support factors and exploration in favour of repetitive individual skill development within teacher-dominated instructional sequences. Detailed analysis of students' algebraic imagery revealed that, while some algebraic forms served a strong and consistent signal function in eliciting meaning and action strategies, others (including simple expressions and tables of values) were associated with unclear signals, frustrating students' abilities to act appropriately in both traditional and computer-based learning situations. These factors acted as impediments to the effective use of mathematical software tools.

At the same time, *strategic* use of appropriate mathematical software (defined as goal-directed, flexible and insightful) supported the development of algebraic skills and understandings in students. Such use was associated for the students with increased manipulative and representational repertoires and increased confidence in their results. The graphical representation was most favoured by all participants, although it was commonly associated with superficial and automatic use. The table of values, while recognised as effective for detailed comparison of functions, was more difficult to interpret and less favoured. Computer algebra tools were found to be most effective in supporting mathematical investigation and the explicit development of extended algebraic processes, such as equation solving.

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LIST OF SOFTWARE

ANUGraph by Neville Smythe and David Ward, Australian National University. Distributed by the Australian Association of Mathematics Teachers.

BiPlane 2.0 Shareware Spreadsheet. Published by Night Diamonds Software.

Calculus T/L II by J. Douglas Childs. Published in Australia by Thomas Nelson Australia.

CC the Calculus Calculator by David Meredith, San Francisco State University.

CoCoA (Commutative Computer Algebra) by Alessandro Giovini and Gianfranco Niesi, Department of Mathematics, University of Genova, Italy.

DERIVE v2.5. Published by the Soft Warehouse, Hawaii, and distributed in Australia through EDSOFT (P.O. Box 314, Blackburn VIC. 3130).

Mathematica Student. Published by Wolfram Research.

Maple V Student Version. Published by Brooks Cole Publishing.

MathMaster 2.21 by Paul Cozza.

Milo™ 1.00 by Ron Avitzur.

Theorist Student Edition. Published by Waterloo Maple.

Xplore - the Mathematical ToolChest by David Meredith. Published by Prentice Hall Australia

One Introduction

“If one changes the tools of thinking available to a child, his mind will have a radically different structure.” (Vygotsky, 1978, p. 126)

Mathematics learning is a complex activity. The thinking which accompanies and directs such learning contains both mathematical and pedagogical elements, as those involved engage in process and content dimensions of the activity, within a variety of learning contexts. At different times, mathematics learning may be seen to incorporate elements indicative of distinct cognitive modes - from sensori-motor (utilising physical movement and goal-directed action) and ikonic (global, intuitive and visual) to the concrete-symbolic and formal modes, more usually associated with mathematical activity across the secondary school and beyond. Of critical importance, however, is a conception of mathematics as a “tool-based activity” (Confrey, 1993a), supported and made possible by such cognitive aids as language (both informal and formal), notation and symbol systems. Additionally, there are external tools for mathematics learning - writing and drawing implements, calculators, geometric construction instruments and, increasingly, computer hardware and software. These last appear to offer new means of transforming, not only the teaching and learning process, but perhaps the nature of mathematics itself (Steen, 1992, Kaput, 1992, Bishop, 1993).

This study of the use of advanced mathematical software by secondary students and preservice teachers attempts to make explicit the ways in which individuals think and act when doing and learning mathematics in a tool-based context. The action occurs within the domain of algebra, long a focal point for mathematics education research. The importance accorded to the learning of algebra is a consistent feature of educational systems worldwide (Wagner and Kieran, 1989, Kieran, 1992); equally consistent is its place as a principal stumbling block for learners. The impact of computer technology upon the twin processes of teaching and learning within the domain of algebra adds a new, potentially explanatory dimension to this problematic field of study.

The research focus lies with the use of mathematical software tools within the context of algebra learning situations. Tools which support manipulative algebra and multiple representations of mathematical objects and processes are now well recognised as means of enhancing mathematical activity (particularly at post-school levels), but they have been little explored as tools for pedagogy and instruction. This study documents the path followed by one teacher (the researcher) as he systematically studies the use of advanced mathematical software applications as tools for both pedagogy and exploration. As a practitioner seeking to improve his own practice through systematic, reflective and (at times) collaborative activity, the author builds upon the research foundation offered by the action research tradition (Lewin, 1946, Carr and Kemmis, 1983, Kemmis and McTaggart, 1988a, 1988b) and blends this approach with the methodological and analytical rigour offered by the Grounded Theory method (Glaser and Strauss, 1967, Strauss and Corbin, 1990).

It is the purpose of this chapter to introduce the principal concerns and direction of this study. Beginning with a consideration of the nature of the research design as a blending of action research and grounded theory, the focus moves from an overview of the role and nature of the new tools now available for mathematics teaching and learning, to an attempt to place the learning of algebra in the secondary school within the context of international, national and state developments since 1980. Attention is then turned to the theoretical bases from which the study initially springs: the theories of learning which underpin the approach taken and the research principles which guided the design. Finally, the unique role of the computer as both focus of study and primary tool for data collection is considered.

As an action research project, the study begins with the identification of a problem. In the present context, this problem revolves around the desire of the researcher as teacher and tutor to “learn to use new tools” - to acquire skills and knowledge of ways in which algebra software tools might best be incorporated into individual learning situations, as tools for pedagogy as well as mathematics. *Doing mathematics* may be considered in terms of an interplay between *grounded activity* and *systematic enquiry* (Confrey, 1993a, pp. 51-54). Mathematical software tools further emphasise this dialectic, as tools naturally linked with action, and yet supporting and encouraging open-ended exploration of mathematical ideas. Mathematics learning may also be situated at various points along a continuum, created by the tension between *instruction* (characterised by teacher-centred activities, in which knowledge is transmitted to the students, who play a largely passive role in the process) and *enquiry* (in which students create meaning from the learning situation as active participants responsible for their

own learning). The role of mathematical software tools within this framework is problematic. It is the purpose of this study to explore the mathematical and pedagogical dimensions of both thinking and action within algebra learning situations, and to make explicit the role of technology within the process.

The project follows a series of distinct action cycles through which the research design is played out and the features and nature of the problem made explicit (Figures 1.1 and 1.2). The various cycles within the design may be recognised as occupying distinct dimensional spaces, aligned with four critical actions which define the study:

- (1) the reflections and activities of the researcher himself, which drive and direct the study;
- (2) observations of individual school students learning to use the available technology as both *mathematical* and *pedagogical* tools within technology-rich algebra tutorial situations;
- (3) collaboration with two groups of preservice teachers, one group focusing particularly upon pedagogical use and the other upon mathematical use of the software tools; and
- (4) the continuing redefinition of the notion of a *technology-rich learning environment*, which provides the context for the study. Such an environment is seen to be defined in terms of three critical variables: *pedagogical content* (the focus of the learning experience), *pedagogical action* (the actions of both teacher and students by which learning is enabled), and the

nature of the tools themselves, which are open to development. In this way, the software tools themselves become an embodiment of the reflections and interpretations of the research process by the researcher. Each of these critical variables provides a basis for revision of the learning environment through the various action cycles of the study.

The overall research design of this study clearly reflects the influence of action research methods, as a practitioner seeking to improve his own practice systematically investigates his own use of algebra software tools, and that of others, through a series of action spirals. Each action cycle begins with the researcher **planning** and **acting** to create a particular interactional situation - what is termed here a *technology-rich algebra learning environment*. **Observing** and **reflecting** upon these observations gives rise to a **revised plan**, embodied in revisions to the curricular content, the pedagogical actions or in physical changes to the available software tools in response to the previous cycle.

Figure 1.1: The Action Research Spiral

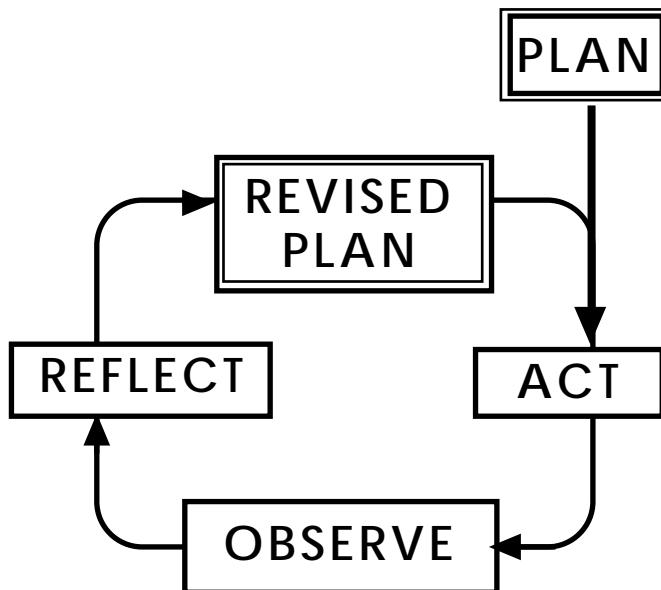
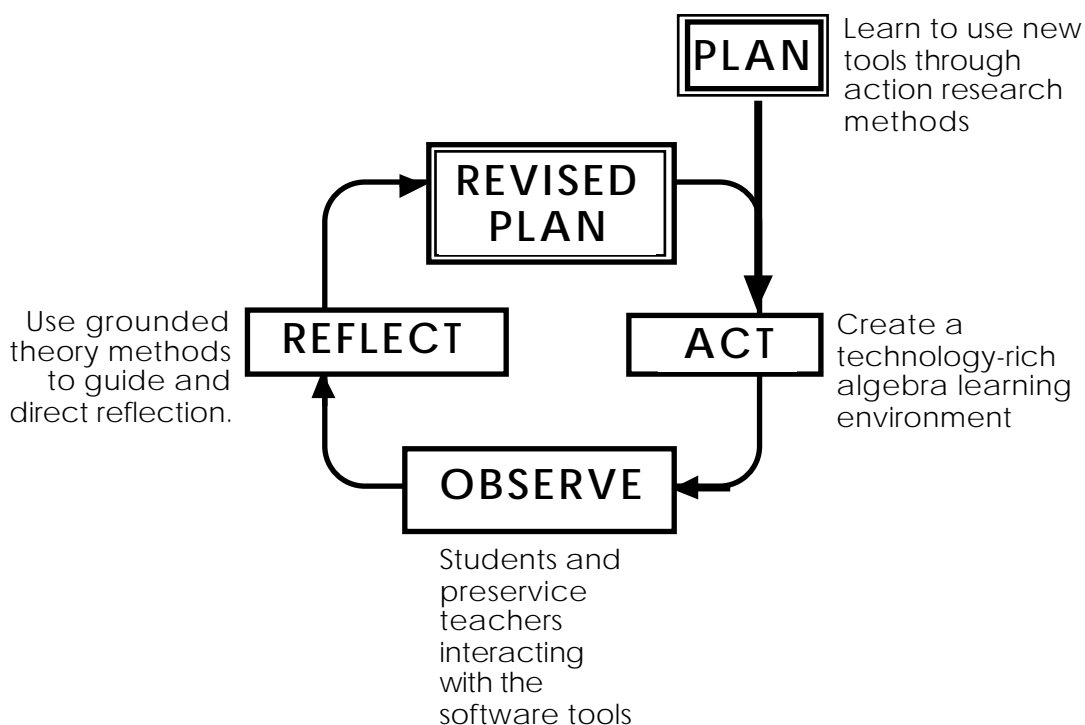


Figure 1.2: The action research design



Miles and Huberman argue convincingly that the weakest link in the chain of qualitative research is that between data and conclusions

(Miles and Huberman, 1984, Huberman and Miles, 1994). Although action research provides a powerful methodological foundation for the study, it offers little to guide the data analysis process. The *grounded theory method* (Glaser and Strauss, 1967, Strauss and Corbin, 1990) has been adopted for this study, offering a detailed and systematic approach to the qualitative research process which provides an ideal complement to the action research methods described above. The use of a grounded theory approach to the analysis of data supports and encourages the development of a theoretical framework which is grounded in the data but potentially applicable beyond the immediate situation. Such a theory is intended to speak directly to practitioners, and to offer guidance and direction to both future research and practice. The merging of action research with grounded theory offers a powerful research design for the current study, clearly defining the reflective act so central to action research.

The focus of a grounded theory design is upon a central phenomenon (or *core category*), a slightly different perspective to the *significant problem* which gives rise to an action research study. By identifying *tool use within algebra learning situations* as the core category, however, the critical link between the two perspectives may be established. The central concern of the researcher in the study which follows, then, is *learning to use new tools*, initially, on a personal level and, subsequently, involving the students and preservice teachers who are drawn to interact with the available software tools within various algebra learning contexts. All participants share this common goal, although each approaches it from a distinct perspective. The unifying phenomenon, or core category, for all participants is that of *tool use*.

In particular, this study explores a phenomenon described here as *strategic software use* - the deliberate, flexible and goal-directed use of available tools as a means of increasing understanding and offering insight into problematic features of the particular learning context. Such use may be seen to represent an ideal, a high level of attainment with regard to both the mathematical learning context and the cognitive tools used within this context - it may be recognised as lying at one end of a continuum along which may be found various styles and dimensions of tool use. The defining characteristics of this style of use, the context within which it occurs and develops, the forms it takes within the interactional process involving user, tool and object of study, and the short- and long-term consequences - all are critical aspects of strategic software use which this study seeks to explicate.

It is hypothesised that thinking and tool use exist within a recursive relationship - each influencing and significantly altering the nature of the other. While this influence is likely to be most evident within a framework of *strategic use*, it is recognised at the outset that such use is likely to be rare. Factors which both encourage and inhibit such strategic use must be made explicit, as must the nature of the use itself, and its consequences.

The research takes as its principal focus the use of a generic software type: symbolic manipulation or, more usually, computer algebra software, by students and student teachers. The use of the software occurs within the context of an algebraic learning environment created for this purpose to provide access and direction in the use of these and other advanced software tools (particularly graph plotting and table of values utilities). The study seeks to document and explain the ways in

which individuals use the various tools available, the context of this use, and aspects of the proposed relationship between tool use and their mathematical and pedagogical thinking. As such, it is interpretative in nature and qualitative in design.

The design of the study and the subsequent gathering of data are driven by four principal research questions. The first two of these define *algebraic thinking* and the third *pedagogical thinking* as the terms are used for the purposes of this project. The fourth defines *tool use* as it is to be considered in the present context.

- What do individuals (researcher, students and preservice teachers) understand by algebra and its components (especially functions, variables, equations, graphs and tables of values) and how might such understandings be related to the use of computer tools?
- What do individuals perceive when they view algebraic objects and how may these perceptions influence their choice and use of available strategies (including the use of mathematical software)?
- What beliefs do individuals bring with them to algebra learning situations concerning the nature of algebra, the ways in which it may best be learned, and the characteristics of successful learning and effective teaching practice? To what extent may such beliefs impact upon the use of technology as a learning strategy?

- Under what conditions do individuals choose to use available software tools, and what forms does this use take? What features of both tool and learning situation serve either to impede or encourage such use?

These themes of algebraic thinking, pedagogical thinking and tool use dictate the concerns and direction of the study which follows. They also preface the structure of the chapters of this book.

The thesis falls into two main sections, Chapters One to Four laying the foundations in terms of the literature, tools and research design, and Chapter Five serving as a bridge, setting out the principal categories by which the data was initially conceptualised and providing insight into both the researcher's approach and beliefs and the subsequent development of a grounded theory of mathematical software use. Chapters Six, Seven and Eight set out in detail the responses of the participants concerning issues associated with algebraic and pedagogical thinking and tool use, as outlined above. Finally, these results are drawn together using the Grounded Theory paradigm in Chapter Nine, leading to the development of a rich and interconnected theory of mathematical software use for the learning of algebra.

New Tools for Mathematics Learning

During the 1970s, the impact of early computer technology made itself felt upon mathematics instruction in the form of hand-held calculators. Soon recognised as offering enormous potential for change in the teaching and learning of mathematics at all levels, there existed at the same time a reluctance on the part of many teachers to freely embrace

such tools. Unsure of how to change their teaching so as to effectively incorporate the new technology, they experienced an initial hesitation concerning the effects and possible dangers. Twenty years later, through extensive research and classroom use, many of the early misgivings have been laid to rest, many of the pitfalls recognised (Hembree and Dessart, 1992). The scientific calculator is now assured of its place as an essential adjunct to the learning of mathematics.

This situation seems destined to repeat itself in the 1990s, not with numerical tools but with algebraic ones. Once more, teachers of mathematics are presented with a dilemma - the availability of technologies capable of enhancing the teaching and learning of their subject, but a lack of insight and clear direction as to how they may most profit from their use. Once more, "(t)he first instinct of educators is to couple the new technology to their old methods of instruction" (Papert, 1980, p. 230). While significant work has centred upon the potential of computer tools such as spreadsheets and graph plotters (particularly through the ongoing Technology-Enriched Algebra Project (Asp, 1991, Asp, Dowsey and Stacey, 1992, 1993a, 1993b) and recent research by Quinlan (1994)) the potential role of manipulative algebra software within secondary school settings remains largely unexplored.

Since 1980, computer software capable of performing all of the algebraic manipulations required for high school and beyond has been available for microcomputers. Initially designed for use by engineers, scientists and research mathematicians, such tools, though powerful, were difficult to learn, and largely unsuited to the secondary classroom. Early "computer algebra systems" have given way to a more sophisticated and appropriate tool in the last five years, capable of the

representation and manipulation of mathematical functions in at least algebraic and graphical forms. Further, recent software has been designed to present these mathematical objects using correct mathematical notation, thus removing the need for students to learn a “computer algebra syntax” in addition to the algebraic syntax already required of them. By allowing the user to select items from menus, templates or palettes, and so to construct and manipulate mathematical expressions with relative ease, such tools at last appear appropriate to incorporate into high school learning situations.

“Enhanced computer algebra software” in its various forms allows the user to represent functions algebraically, graphically and numerically; ideally the user may move interactively between these representations. Additionally, some manipulation of these forms is possible. The algebraic representation, for example, may be rearranged, expanded, simplified and, in some cases, factorised. It may be differentiated and integrated, numerically and often symbolically. Exact arithmetic calculations are often possible, allowing work with fractions, surds and complex numbers in exact forms, or to any desired degree of accuracy. The graphical form may be rescaled by adjusting the “viewing window”; “zooming” in or out to allow the function to be observed in a wider context, or in finer detail. Points of interest may be isolated and identified. Equations may be solved graphically or algebraically. The power of computer algebra is linked to the versatility of the graph plotter. The computer becomes a means of exploring the nature and properties of the functions which make up the larger part of the study of mathematics at the higher levels, and of relieving much of the syntactical burden which proves a barrier for many learning algebra.

The availability of such tools presents enormous implications for change in the ways in which mathematics may be learned. Much of the time spent in high school mathematics courses currently arises from the perceived need for students to acquire relative mastery of the skills of algebraic manipulation. The increasing availability of computer tools which will perform these manipulations more quickly and accurately than was previously possible suggests that such time may be spent more profitably in other ways. In particular, reducing the time spent on the mastery of manipulative skills may allow more time to be spent on activities which will aid in concept development and understanding, and on the applications of mathematics to “real-world” situations (Heid, 1988, 1989). Such features appear sadly lacking from most senior mathematics courses, both here and overseas (Tobin and Fraser, 1988, pp. 77-79).

The use of such computer technology permits three fundamental advantages which would otherwise be too difficult, or quite impossible to achieve within more traditional mathematics learning situations:

- *New Modes of Representation:* The use of computer tools facilitates a variety of representations which may serve to deepen understanding and encourage investigation.
- *Power of Calculation:* Computer tools are capable of performing both numerical and symbolic manipulations quickly and accurately, freeing students to focus upon the context and meaning of the problem situation, and to investigate applications which might otherwise be too difficult or inaccessible to them.
- *Interactivity:* Computers are dynamic tools, quite distinct from the passive aids which characterise many school experiences.

The ease with which the student may direct and control the computer, accessing powerful mathematical features further justifies their inclusion in the curriculum.

The parallels with the advent of hand-held calculators are precise. Activities devoted to particular skill development, such as the long division algorithm, were found to require a disproportionate amount of class time. The decreased emphasis upon such skills which has resulted from the widespread use and availability of calculators has freed such time for other purposes. It seems likely that tools capable of algebraic manipulation and the multiple representation of functions may also become a means of freeing up the curriculum for activities determined to be more appropriate than the repetitive skill development which is currently the norm.

Finally, such tools call into question existing assumptions regarding the nature of teaching and learning in mathematics. Instructional practices which have been considered successful and, in many cases, “expert”, are being challenged by this technology. All too often, such practices are being exposed as encouraging rote learning and superficial understandings (see, for example, Schoenfeld, 1988). Those who have relied upon these practices may well feel threatened by recent calls to incorporate the new technology as one means of increasing the emphasis upon understanding and conceptual development in mathematics learning.

This study potentially offers new knowledge of the ways in which those learning algebra engage in the use of available software tools. Such considerations are critical in seeking to understand ways in which such

tools may be effectively used as means to increase understanding and enhance algebra instruction at all levels.

Algebraic Thinking in Secondary Schools

The teaching and learning of algebra in the secondary school have become objects of increasing attention worldwide over the past decade. The beginning of the 1980s appeared to be a time of great resolve and desire for change in the international mathematics education community, typified by the large-scale *Concepts in Secondary Mathematics and Science Project* (C.S.M.S., 1980-82) and the *Cockcroft Report* (1982) in Great Britain, and the National Council of Teachers of Mathematics (N.C.T.M.) *Agenda for Action* (1980) in the United States. More recently, the release of the N.C.T.M. *Curriculum and Evaluation Standards* (N.C.T.M., 1989) and, here in Australia, the Australian Education Council's documents, *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1990) and the subsequent *National Mathematics Profiles* (Curriculum Corporation, 1993) suggest that the momentum has been sustained. Australian work on "concrete approaches to algebra" (Booth, 1989, Quinlan et al, 1989 and Pegg and Redden, 1990) appear to be indicative of recent research efforts. Certainly, with their inclusion in recently released Syllabus documents in New South Wales and Queensland, and consequently their inclusion in the new generation of textbooks for junior mathematics, there are hopeful signs that teachers may begin to move away from the traditional "rote learning" approach to the teaching of algebra.

The *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1990, pp. 187-189) describes algebra at all levels of schooling as serving three fundamental purposes:

- *Expressing generality* : As the primary means by which we may describe and understand patterns and generalisations in a multitude of situations.
- *Describing Functions* : Providing the means by which functional relationships may be defined and represented using a variety of forms, especially numerical, graphical and symbolic.
- *Solving Equations* : These form the essential mathematical problem solving tools by which problem situations may be analysed and described mathematically, and then redefined in a simpler or more accessible form.

Distinguishing between the “syntax” of algebra, in terms of the rules and manipulations which govern its use, and the “semantics” (the meaning and context of algebraic activities), school mathematics has been dominated by the former. A critical pedagogical role of computer algebra, then, may lie in its ability to perform the syntactical operations of algebra, and so to allow greater emphasis upon context, meaning and applications. Used appropriately, it should also permit students increased opportunities for reflection upon the processes involved, rather than being absorbed by the manipulations themselves.

While there are grounds for optimism that algebra learning in the junior school may be expected to become more concrete and, hopefully, more meaningful for students, there appears to be little of the same pressure at the higher levels. Middle school mathematics courses (Years 9 and 10) are still dominated by skills of manipulation, largely in preparation for the requirements of the calculus in the senior school. Students attempting the calculus-based courses in Years 11 and 12 are expected to be proficient in algebraic manipulation, possibly to the exclusion of other skills. Students without solid grounding in this area may not be expected to succeed, even at the “average” senior level (which, in New South Wales, is the 2 Unit Mathematics course).

With retention rates into senior education in Australian schools reaching 80% and with ever-increasing requirements for tertiary entrance and employment, schools are feeling growing pressures to admit students to such courses who would not previously have been considered. Enrolment patterns in senior mathematics courses offered in New South Wales over the period from 1983-93 may be considered representative of those in other Australian states, and are depicted in Figures 1.3, 1.4 and 1.5 (see Appendix A for the corresponding numerical information). The graphs illustrate the percentages of the total student candidature for that year attempting the four courses available (Figure 1.3), the actual numbers of students for each course (Figure 1.4) and, finally, the percentage increase for each course over the eleven year period.

Figure 1.3: NSW HSC Mathematics Courses 1983-93: Percentage of Total Candidature

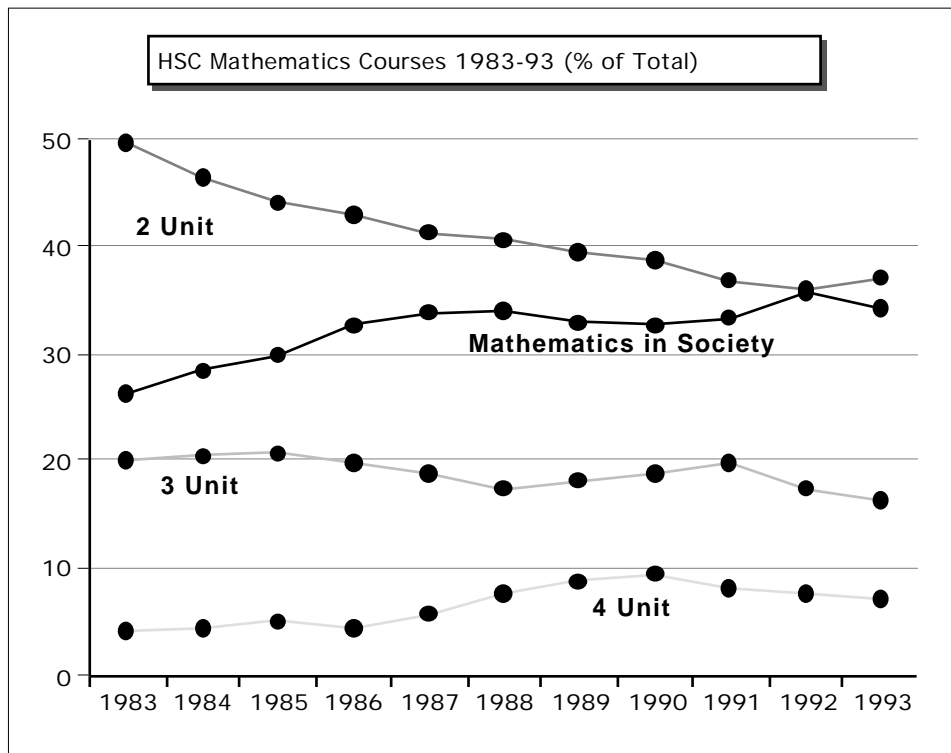


Figure 1.4: NSW HSC Mathematics 1983-93: Numbers of Candidates

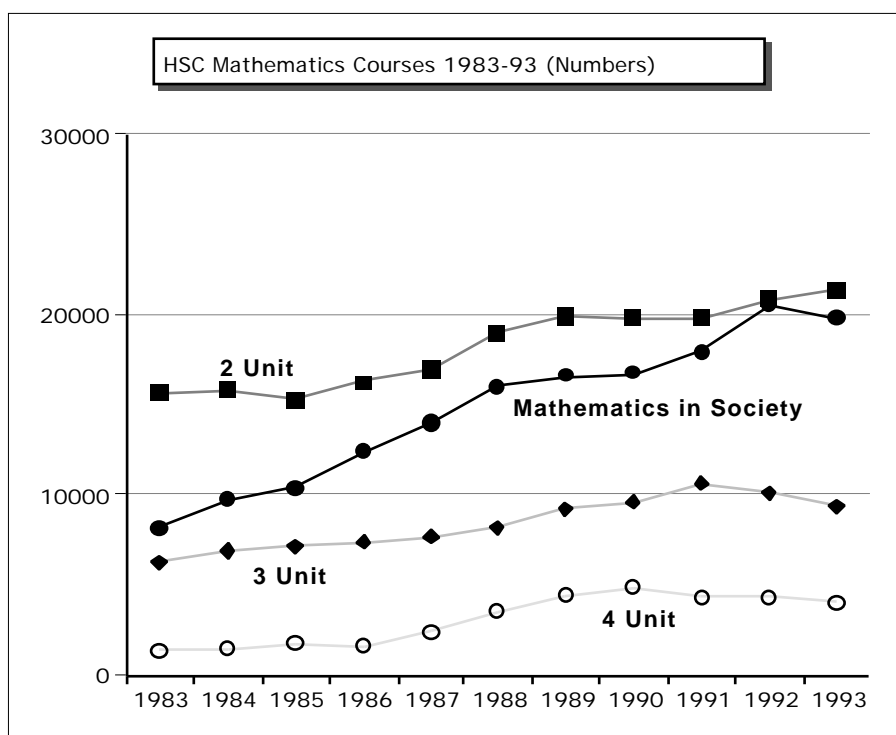
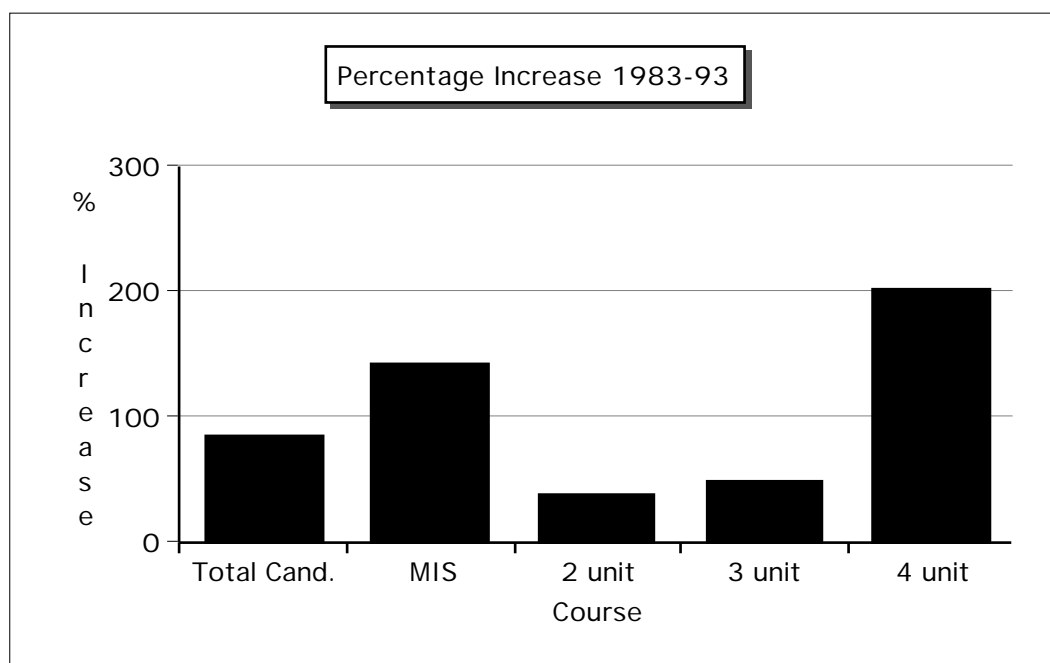


Figure 1.5: NSW HSC Mathematics Courses 1983-93:
Percentage Increase



Of the four courses offered, the *Mathematics in Society* course is a non-calculus option, considered as a terminating course for purposes of tertiary mathematics study. (A fifth course was offered for the first time in 1991, intended for students at a lower level than that catered for by the *Mathematics in Society* course. This course, *Mathematics in Practice*, accounted for only about 5% of the candidature in 1993.) Those courses labelled *2 unit*, *3 unit* and *4 unit* are provided as tertiary preparation courses for mathematics, and are characterised by increasing breadth and depth of content. Of critical significance with regard to interpretation of these graphs is the large increase in the total candidature in mathematics for the Higher School Certificate over this period, from 31,448 students in 1983 to 57,709 in 1993. This is indicative of the increasing enrolments across all subject areas in the senior years over this period.

Within the context of this increase in overall numbers, the graphs reveal significant changes in enrolment patterns. The greatest increase over this period has occurred in the *Mathematics in Society* course. Although the *2 Unit* course still accounts for the highest overall proportion, it has dropped significantly in this regard. Clearly, the majority of the “new” students (those who would probably not have continued on to senior study in 1983) are enrolling in the less demanding course. At the same time, the actual numbers of students in all courses have increased - there are physically more students attempting all mathematics courses than there were a decade ago. Interestingly, the greatest increase overall (when balanced against the increased student population) has occurred in the highest course, the *4 Unit* course. It seems likely that syllabus changes to this course in 1986 served to make it more accessible than previously; it has also benefited from a significant scaling advantage when scores for tertiary entrance are computed.

Within the secondary school, algebra has traditionally served as a “gatekeeper” for study at the higher levels, performing in many instances a deliberately discriminatory role in separating students. Just as a lack of proficiency in numerical skills may serve to deny many students opportunities to engage in mathematical content areas available to their peers, so too does poor algebraic facility. As hand-held calculators have served a scaffolding role, supporting students in their study of these aspects of mathematics previously denied them, so too may algebra software expand the options for many, not just in the senior years, but across the secondary school and perhaps beyond. Questions as to the effectiveness and potential of advanced

mathematical software tools in the learning of algebra, then, become both educationally and socially relevant.

Theoretical Frameworks

In seeking to describe and understand the ways in which students and teachers think about and use advanced mathematical software, and the consequent effects of such use, the role of theory is perceived to be one of providing initial guidance and direction in a fledgling field of enquiry. In the model developed for this study, the computer is perceived as being, at the same time, both focus and method of enquiry. Advanced mathematical software, as it is used by individuals involved in mathematics learning, provides the principal target for description and explanation. As the primary mode of data collection in the research design, the computer, too, becomes the central means of enquiry, as it provides an ideal tool by which such interactions may be made explicit.

No single theoretical model appears sufficient to provide description and explanation at a suitably rich and meaningful level. Rather, a melding of several compatible and complementary theories of learning and cognition allows the complex interactions of individual and technology to be categorised in ways which illuminate different aspects of the process. Thus, at what might be considered the “micro” level, the SOLO taxonomy (Biggs and Collis, 1982, 1991, Collis and Biggs, 1991, Collis, Watson and Campbell, 1992) provides a detailed descriptor, not only of the various modes of thinking which characterise the cognitive activities of different individuals at different times, but also of the developmental sequence occurring *within* each of these modes. The taxonomy fails, however, to provide adequate explanation for the ways

in which learners move between the various modes of thinking, and the potential role of teacher (and technology) in assisting such transition.

A theory of learning proposed by Pierre van Hiele (1986), specific to mathematics education, offers a different perspective on the modes of thinking, but one which is consistent with that of the SOLO taxonomy. There is, in this way, the possibility for an increased richness of description as each theory illuminates the same aspects from a different angle, each casting light upon features which the other may fail to fully accentuate. Further, van Hiele is concerned primarily with the role of the teaching process in the transition between levels of thinking, and offers much in the search to explain the ways in which teacher and technology may work together in facilitating student learning and understanding. The van Hiele theory, then, offers a means to paint with broader brush strokes, observing and seeking to explain the movement across modes of thought and styles of learning.

Although both the SOLO Taxonomy and the work of the van Hieles were inspired by the ideas of Piaget, both perspectives recognise the critical importance of the learning *context* in seeking to understand the learner. The van Hiele theory goes further still, considering the process by which appropriate teaching may contribute to cognitive development, and the critical role of language in the learner process. In this respect, the work of van Hiele has appeared to move closer to that inspired by the Soviet psychologist, Lev Semanovich Vygotsky (1962, 1978, 1987). These theories consider learner as inseparable from context, particularly the social and cultural context in which learning occurs. Vygotsky believed that all higher cognitive processes are acquired initially through social interaction - occurring on an inter-personal level

before they become internalised to occur on an intra-personal level. Although he made no specific mention of the role of computer technology (since such technology did not exist in the early years of this century when his theories were developed), Vygotsky's views appear to offer much which inform a consideration of the ways in which individual learning may be enhanced by the use of suitable tools in appropriate contexts. Further developments of Vygotsky's work by Bruner (1968, 1986) and Wood (1980, 1986) provide means for detailed analysis of individual learning within collaborative and tutorial situations, particularly relevant to the present study.

Finally, as a means of describing and understanding the ways in which people learn, and ways in which people learn mathematics in particular, principles of constructivism provide a firm and broad foundation. From a constructivist perspective, learning is not achieved through the transmission of knowledge from teacher to students; rather, each student constructs his or her own meaning from the learning experiences encountered, meaning which undergoes a process of personal and social negotiation before it is internalised. Traditional exposition models of instruction assumed that the same message was received by the thirty or so different students to whom it was transmitted; constructivism denies the likelihood of such uniformity. A single instructional message may well be interpreted in many different ways. The teacher's role must still involve providing meaningful and carefully planned learning experiences; of even greater importance, however, becomes the responsibility for providing the means by which students may question what they have experienced, may compare and contrast their perceptions with others (especially their peers) and may

negotiate meaning which is consistent with their existing understandings and with those intended by the teacher.

When viewed from the Piagetian perspective of constructivism, the process of learning becomes a spiral of equilibrium -> disequilibrium -> re-equilibrium. The role of the teacher, in such a view, is to attempt to induce “perturbations”, creating just enough disequilibrium in the learners to encourage re-equilibrium (Doll, 1986):

The teacher must intentionally cause enough chaos to motivate the student to reorganise. Obviously this is a tricky task. Too much chaos will lead to disruption (Bruner, 1973, Chapter 4), while too little chaos will produce no reorganisation. (p.15)

The traditional “sequence” of instruction assumes learning to be linear and common to all learners within the group. A constructivist view, however, presents human learning as complex and branching, not simple and linear. Individuals learn in different ways: not all at the same times, nor in the same straight lines. The powerful branching features available through a program such as *HyperCard* makes such learning a very real possibility within mathematics learning situations. Students working through instructional sequences developed in this way might be expected to do so in different ways, at different rates, and to make different decisions along the way, concerning their style of investigation, and their path through such a program.

A Blending of Research Approaches

There are some who might argue that the present study is inappropriately classed as action research. Educational action research as it has been practised here in Australia has been increasingly

identified with that practised by those associated with Deakin University from the early 1980s to the present time (Kemmis and McTaggart, 1988a, 1988b). Arising from a sense of frustration with “the distance between the exigencies of practice and psychometric approaches to educational research” (McTaggart, 1991b, p. 44) and wanting to “work more closely with teachers, consultants, parents and students” (McTaggart, 1991b, p. 44), their interpretation of action research borrowed heavily from critical social science in “trying to make schools and systems more reasonable, more just and more humane for students” (McTaggart, 1991b, p. 44).

The institutional focus implied in this view was quite deliberate. McTaggart notes disparagingly that “some educators working alone and following the technical imperatives of ‘the action research spiral’ felt they were doing action research” (McTaggart, 1991b). She goes on to state that

as our work focused on the theoretical development of the rationale for action research... the need to specify minimum requirements for action research - the axiomatics of action research praxis - became paramount... In order to develop the rationale and purposes of action research, finer distinctions became necessary, and the language of action research became more sophisticated, and more conscious of concepts drawn from social theory (p. 44).

The present study appears by such a definition to lie outside the realm of “legitimate” action research, damned by its individual focus and its non-institutional context. The very notion of “legitimate” action research, however, appears contrary to the fundamental principles by which it was conceived, and critics of this view, such as Gore (in McTaggart, 1993a, p. 43), “claim that certain specialist discourses have occupied the action research ground and have served to disempower ‘practitioners’”. As Gore herself notes (Gore, 1991),

'Action research' as understood in teacher education circles, connotes specific practices. Given the wide range of practices that go by the name, however, it is clear that the term has no meaning outside its construction in particular discourses. If this statement is accepted, then rhetorical attempts to reserve the label for a particular set of practices ... are predestined to fail, functioning instead to police discursive boundaries (p. 47).

It is possible that the focus upon theoretical principles, the increasingly sophisticated language drawn from social theory and the perceived need to specify minimum requirements for action research described by McTaggart above may well have had the effect of distancing it from the very practitioners for whom action research was created. Denying any group ownership of a research method, the present study lays firm claim to the label, action research. While individual in focus and occurring outside a formal institutional milieu, nonetheless the primary concerns of practitioner seeking to improve his own practice through systematic application of cycles of action, observation and reflection justify such a claim.

Giving form and direction to the reflective act, and central to the analysis of data gathered for this study is the *grounded theory* method, in which the primary purpose of the research design is the generation of theory which is developed directly from the data - in this case, a theory of mathematical software use in the context of algebra learning. As defined by Strauss and Corbin (1990):

... (a) **grounded theory** is one that is inductively derived from the study of the phenomenon it represents. That is, it is discovered, developed, and provisionally verified through systematic data collection and analysis of data pertaining to that phenomenon. (p. 23)

The originators of the grounded theory approach defined four central criteria by which such theory may be judged (Glaser and Strauss, 1967, pp. 237-250): **fit** (the extent to which the theory reflects and is faithful

to the reality expressed in the data); **understanding** (the extent to which it reflects the realities of the practitioners and is congruent with their perceptions); **generality** (in the sense that the theory should be abstract enough to be applicable to a variety of contexts appropriate to the phenomenon), and **control** (being the measure of appropriate action towards the phenomenon which flows from and is directed by the theory) (Strauss and Corbin, 1990, p. 23).

The theory generated by this approach may be expected to fulfil traditional empirical requirements of significance, reliability and validity, arising as it does from a systematic analysis of the reality as reflected in the data. Through detailed and extensive analysis of both the categories by which the phenomenon may be recognised and, more importantly, the network of relationships which link these, the approach may be expected to yield results which are conceptually dense and rich in both descriptive and predictive power. Again, from Corbin and Strauss (1990):

Its systematic techniques and procedures of analysis enable the researcher to develop a substantive theory that meets the criteria for doing 'good' science: significance, theory-observation compatibility, generalisability, reproducibility, precision, rigour and verification. (p. 31)

Lying at the heart of the Grounded Theory method is what the authors refer to as "the Paradigm Model", a method by which subcategories may be linked to a category in a set of relationships which place it into a rich and well-defined analytical context (Strauss and Corbin, 1990, p. 99). In simplified form (and within the context of the present study) this model may be represented as:

- (a) CAUSAL CONDITIONS -> Events leading to occurrence of...
- (b) PHENOMENON ->..... Using the software...
- (c) CONTEXT -> The algebra learning situation...
- (d) INTERVENING CONDITIONS -> Impediments and Imperatives...
- (e) ACTION/INTERACTION STRATEGIES-> Form of the interaction...
- (f) CONSEQUENCES ->..... What are the results?

Mapping the major categories of the study in these terms forms the basis for more intensive theoretical interpretation of the data. Ideally suited to this task is the qualitative analysis software tool, *NUD•IST* (Richards and Richards, 1993) which encourages and supports the creation of logical trees and networks of relationships to form an *index system* by which the analysis may be defined. The program appears to offer an ideal complement to the grounded theory approach, supporting the detailed and yet extensive analysis demanded for the generation of substantive theory. *NUD•IST* is used in the present study to initially support the coding of data using broad general categories (*People, Words, Actions, Tools and Content*). These broad categories reflect the initial conception of the project on the part of the researcher. Increasingly fine detail in the coding is then facilitated by the retrieval capabilities of the program, and the “Index System” develops as a direct result of the analysis of the data. Finally, categories and sub-categories may be moved easily to reflect the growth towards a theoretical structure which results from the application of Grounded Theory procedures. More detailed description of this process is provided in Chapters Four and Five.

The study centres upon the **core category** of *mathematical software use*. It is the primary purpose of the project to describe, explore and

explain this phenomenon within the context of algebra learning situations and so to learn to use these new tools as supports for effective learning. **Causal conditions** for this phenomenon will include features of the particular algebra learning situation which lead to the use (or non-use) of available software tools. These may be recognised as particular events or conditions which give rise to specific interactions between users and tools.

The **context** is concerned with the conditions under which the phenomenon manifests itself. Characteristics of the user and the learning situation are critical. **Intervening conditions** are broader, more general aspects which influence the interactions. These may serve as either *impediments* or *imperatives* (hindering or encouraging tool use).

The actual tool use may be considered in terms of **action/interaction strategies** and these taken together with the various antecedent conditions will help to determine the particular **consequences** which result. Analysis in these terms will be applied to, not only the central phenomenon, but each of the major categories and subcategories which arise from the data. In this way, the generation of theory will be both intensive and extensive in relation to the phenomenon in question.

The Computer as a Research Environment

Seymour Papert, the creator of *LOGO*, once described the computer as the “proteus of machines”, with the potential to be all things to all people (1980). While such a claim is somewhat expansive, within the domain of educational research, and particularly with regard to

research of the type suggested above, the computer offers unique and exciting possibilities. When used within a flexible programming environment such as that offered by *HyperCard*, it becomes possible to use the computer as a means of studying itself - or at least the interactions of teachers and students with it.

As a means of capturing the interactions of individuals with available technology, a unique research instrument was developed for this study. Based in *HyperCard*, the program consists of three major components:

- (1) A series of interactive instructional modules, spanning content and processes across the secondary years, within which computer algebra, graph plotting and tables of values tools are available, and their use encouraged. The program provides immediate access to external software tools which accompany and extend the instructional process, encouraging free exploration of the ideas and processes under consideration.
- (2) In addition to the external software tools, a “mathematical toolkit” (called the *MathPalette*) is provided within the program, making available an extensive range of supportive functions which include graph plotting, tables of values, equation solving, coordinate geometry features (midpoint, gradient, distance and equation of a line through given points), numerical substitution, derivatives and areas under curves. The usual symbolic, graphical and numerical representations by which algebra learning is increasingly enhanced are supplemented within the program by an

interactive “concrete algebra” mode, by which algebraic expressions and equations may be created and acted upon using concrete representations. These facilities are intended to further support and encourage open-ended exploration of algebraic ideas and processes within the context of the instructional modules and beyond.

- (3) Additional to these mathematical components of the program are specific research components, designed to generate appropriate research data related to thinking and tool use.

The “research questions” which accompany the instructional modules consist of a series of specific tasks, intended to reveal aspects of mathematical and pedagogical thinking on the parts of those using the program. These tasks range from open-ended “Grand Tour” questions (Spradley, 1980) such as “Describe a typical maths lesson” to the Likert-style *Constructivist Learning Environment Scale* (Taylor and Fraser, 1991). Cards portraying a range of visual algebraic images are used to generate verbal responses; the cards are then grouped by participants (after Stein, Baxter and Leinhardt, 1990). In several cases, these images are then used as the basis for a Repertory Grid analysis (after Kelly, 1955), potentially providing further insight into individual thinking and understanding of algebra. A simple attitude scale is also included, adapted from Quinlan (1992). Each of the various research components designed for the purposes of this project have been adapted to a computer-based format using *HyperCard*, allowing entry of data in a format consistent with that of the overall research design.

The “research version” of the program has been enhanced through the addition of three features, labelled “comments”, “prompts” and “probes”. At any time during a session, the user is encouraged to make comments regarding the program, the ideas encountered, their own responses, and so on. These comments are recorded along with the other research data from that session: which cards are viewed, which buttons are pressed, which functions are entered, graphed or analysed in other ways; times for each of these activities are also recorded. “Prompts” are questions posed within the context of the instructional module, which the user is expected to answer. They may also take the form of suggestions as to the possibilities for the use of computer algebra, or graph plotting or table of values tools at particular points. Finally, “probes” have been added at certain critical points in the modules. When users access a computer algebra tool, or graph plotter, or table of values, for example, they will be asked afterwards what their intentions were in this regard, and how effective they found the tool to be. When first selecting particular modules, such as the “Beginning Algebra” module, student teachers will be probed regarding the way in which they would sequence an introduction to algebra themselves at present; this may be compared with a similar probe at the conclusion of the module. Students are probed as to their understanding of key concepts, such as “function”, “variable” and “equation”, in addition to their attitudes towards mathematics, and their own assessment of their abilities in this regard. The research tool, then, provides a means, within an instructional framework, of studying the interactions of students and student teachers with mathematical software tools.

The research design is that of case study, focused upon individuals working through the instructional materials, and providing additional

research data related to preferred images and representations of algebraic concepts, facility and understanding of algebraic manipulations, as well as attitudes and beliefs concerning pedagogical factors related to algebra learning. Responses are then coded and analysed using qualitative research software (*NUD•IST*) allowing regularities and relationships within the data to be observed and explained. Further, *NUD•IST* supports, not only the retrieval of data according to the codes and categories applied, but also the development of the *Index System* which defines the analysis of the data. While coding begins at a general level, it becomes increasingly refined as the data retrieved from each general code is recoded in finer detail, consistent with the grounded theory approach. Finally, these categories and sub-categories may be moved around and positioned in relation to each other, allowing the building up of a complex and intricate theoretical structure which becomes the grounded theory itself.

The grounded theory approach has been applied to the analysis of data in order to offer findings which are integrated, detailed and rich in explanatory power. The theory of *mathematical software use* which this study explores is intended to offer both detailed explanation and generalisability within the confines of the chosen research design. The overarching action research design further supports the ongoing development of a computer-based technology-rich learning environment for the learning of algebra across the secondary school years. It is anticipated that these twin outcomes of the research design will offer guidance and direction to teachers who would use mathematical software tools as means to enhance algebra learning within other settings.