

Seven *Thinking about Learning*

If the primary units of analysis for the previous chapter were definitions and images, then this chapter is principally a study of beliefs. As Chapter Six sought to build up profiles of algebraic thinking by the various participants, so does this chapter examine aspects of pedagogical thinking by the students, preservice teachers and by the researcher. Beliefs and attitudes regarding the learning of mathematics must feature critically in understanding the responses of individuals to computer technology as a medium for algebra learning. This chapter examines beliefs and perceptions concerning:

- how algebra is best learned;
- features of a typical mathematics lesson;
- images and metaphors associated with more and less effective teaching practice, and
- constructivism in algebra learning.

From this data a second set of individual profiles may be developed which will further inform our consideration of algebra learning within a technology-rich environment.

How Do You Best Learn Algebra?

Throughout the entire data collection process, the question which became associated with the major theme of the study was the one which greeted participants each time they entered the computer-based instructional modules: *“How would you describe your beliefs about algebra and the ways you best learn it?”* The responses of the participants provide valuable insights into both their perceptions of effective learning and the nature of mathematics itself.

Stephen offered his responses to this key question several times over the period of the study:

1. I best learn algebra by repeating questions to understand why you do the question a particular way.
2. [Success in mathematics follows from] an understanding of what you want to find, how to go about finding the answer, [and] understanding why you use particular steps and formulas.
3. The best way to learn algebra is to see a diagram of the figure or graph and working out methods to find answers using rules and formulas.
4. The way I best learn algebra is in seeing graphs or curves of the equation and being showed how the rule is derived.
5. A successful maths lesson is when we were allowed to study and look over our work. The teacher would come around and help us with our problems and how to set out our answers. A student would help another and show how he approached the problem.
6. I learn it best by seeing the graph of an equation you have to work out and by knowing why you have to use a formula.

These comments spanned the two year period of Stephen’s involvement with the study, from his commencement of Year 11 to the end of his Year 12 studies. Six critical elements may be discerned as characterising Stephen’s beliefs about algebra learning:

1. Visualisation (especially using graphs),
2. “Being shown” how to arrive at a solution,
3. Repetition,
4. “Working out” a method for arriving at an answer,

5. “Knowing why” you use rules and formulas, and
6. Algebra is associated with memorising rules and formulas, and finding answers to problems.

The first two elements may be associated with a **passive** approach to learning, the second two with a more **active** involvement. All suggest common themes which may be observed to greater or lesser extents across the sample, from students to preservice teachers. In Stephen’s case, the role of visualisation is clearly a dominant one, occurring as a repeated theme in several statements. The clear distinction between his first statement and the subsequent responses suggests a growth in metacognitive awareness, which probably arises as a direct result of his involvement in the research program. The regular requests for analysis of his own thinking about algebra and learning served to make him aware of his own learning and encouraged him to refine his response. The similarities of the latter responses suggest that his thinking on this matter had become relatively stable and that the long term involvement within a technologically rich learning environment may have done little to alter his thinking about learning other than to impress upon him the value of visualisation as a learning tool.

Stephen’s reference to “knowing why” suggests that he perceives **understanding** as a central factor contributing to an effective learning experience. Within the context of his view of algebra as consisting of formula- and rule-based methods for finding answers to “problems”, however, his “understanding” may be more instrumental than relational.

Finally, the perceived role of the teacher in effective learning is likely to be significant. Explicitly, successful learning was associated with a more student-centred learning context, in which the teacher assumed a supportive role. Throughout Stephen's responses, however, the teacher is never far from consideration - as the one who "shows how the rule is derived", who provides the reasons for the use of rules and formulas and, fundamentally, as the primary source of the "problems" by which algebra itself is defined. A potential conflict may be discerned between Stephen's association of successful learning with student-centred methods and yet his reliance upon the teacher as source of both that which is to be studied and the ways in which this study may best be achieved.

Stephen's peer, Ben, offered the following responses over a similarly protracted period of involvement within the research program:

1. I best learn algebra by revising the work. I've just got to chisel it into my memory because of the formulas.
2. [Success in working with functions and variables follows from] full understanding of what you are doing.
3. [Success in coordinate geometry follows from] good visualisation of what you are working out mathematically. Good understanding of the formulas.
4. I learn [algebra] best by continually just doing algebraic problems, until I get used to it, and start seeing different methods, different ways to go about it.
5. [A successful lesson was one in which] the teacher was able to get across the true mathematics and to help me to fully understand how to answer all the questions used throughout the lesson. By true mathematics I mean the substance behind it, why it's meant to do what it does.
6. [The three most important factors in learning algebra are] understanding, ability, memory for rules concerning algebra.

Once again, the common themes are those of repetition, understanding (but within a context of memorisation of rules), visualisation and teacher dominance. Once again, the teacher is perceived as the source of knowledge and skills which must be learned. There is an implication,

too, in the last response that ability is a factor in determining success in algebra - that some people are naturally better than others.

Andrea offered two comments regarding algebra learning:

1. The best way to learn algebra is to memorise the equations and also know their purpose and use.
2. The best way to learn algebra is to learn the basic ideas first then take a few challenging questions according to the individual's level.

Present once again are the themes of understanding linked with memorisation and individual ability as a deciding factor in algebraic success. The second response, however, suggests some change in perception over the intervening period since the first. It appears to reflect elements of the van Hiele **stages of learning**, which begin with information and guided orientation, and progress later to free orientation which might be associated with Andrea's notion of challenge. Since these stages of learning provided the basis for the construction of the algebraic learning environment for this study, it does seem likely that she has adopted elements through her involvement with the program. Andrea's perception of challenge is notably absent from the views of the other respondents.

Like her conception of algebra, Jane's view of effective learning remained stable over the period of the study, centred upon repetition and memorisation.

1. ...doing lots of questions and examples that are given to me.
2. ...practise and revision.
3. ...revision, learning rules.
4. ...common sense, knowledge, and to use the skills you know.
5. ...you learn [algebra] best by revision, practise.
6. ...revision, practise and study.
7. I find it better when I work with the people around me. I like to have the time to think about what I am doing.
8. Revision, common sense and practice.

Jane's view of algebra learning appears to be largely passive, dependent upon the teacher, although she acknowledges that she works best in a social context.

For Tony, the element of innate ability is an immediate one, along with the now familiar themes of memorisation, understanding and repetition within a teacher-dependent learning context:

1. I think that some kids can achieve high scores in algebra right away and some can't.
2. [Success results from] knowing what the pronumerals and numbers stand for.
3. Learn all the formulas that go with [algebra].
4. Learning the rules involved and working it out.
5. [The three most important things for success in mathematics are] learning the rules and the formulas, studying hard and doing all your homework.
6. I best learn algebra by using the computer to show things graphically. This is better than when teachers blab on and write stuff on the board. The computer is most helpful in showing us alternate ways to do things and showing you graphically how to do the sums. The computer is least helpful when it crashes! When you get something wrong, the computer doesn't really help you - it can't work out what you did wrong. I like using a computer algebra program because it helps me to see what I have done wrong after I have done it.

Like Stephen, visualisation is a feature of Tony's learning of algebra; like Ben, Tony recognises the advantages of encountering **alternative** approaches to concepts and methods. The computer figures strongly in Tony's perception of effective learning, reflecting his experience with algebra software tools. He notes disadvantages as well as advantages in the use of technology in this context.

Patrick's experience of algebra learning had largely involved the use of computer-based concrete manipulatives, and some limited use of computer algebra software. Like Tony, he sees innate ability as a significant factor in deciding success or failure in algebra learning, but he reacts positively to the concrete (visual) approach:

1. I think it is important to be able to understand algebra to do it.
2. ...it is best to learn it at your own pace because taking it too fast could ruin your image of algebra.
3. [Success in algebra follows from] trying different kinds and using the knowledge of those who offer assistance to you.
4. The best way to learn it is not imagery (in your head) but in visual maths, with objects in front of you.
5. To be successful in algebra you must KNOW what you're doing and BELIEVE that you can actually do it. When I used the computer to solve some equations I didn't learn anything because the computer did it for me leaving me only with an answer not the knowledge of how to do it.
6. Algebra is using maths in a different way and it is best learned by using solid and visual objects.
7. You need to be taught well the first time around or you won't understand as well when older. You should have visual objects and you should be good at numbers.

Patrick offers some unique perspectives, in addition to echoing the themes already observed. He notes, for example, that the **pace** of instruction is a significant factor, and that **confidence** in your own abilities influences learning. He observes that skill in numeracy is an important precursor to algebraic facility.

The view of algebra which emerges from the responses of the six students in the study is one which is surprisingly consistent. Successful algebra learning, for all students, is associated with **understanding**, but this appears to be in the sense of “knowing what to do” rather than “knowing why it is done”. This form of understanding for the students is perceived as resulting from repetition, leading to memorisation of rules and formulas. While it may result from the efforts of the student (“working out”) it appears more likely to involve teacher intervention, in which the student plays a largely passive role (“being shown”). Associated with the passive role of “being shown” appears to be strong general reliance upon visualisation and frequent reference to the graphical representation of algebraic ideas. Although the teacher is generally recognised as the source of both knowledge and method in algebra learning, students also commonly make reference to a

preference for working within a social context, learning through interaction with their peers.

The preservice teachers appeared to interpret understanding differently from the students. While the latter group saw it as following from the learning of rules and formulas, the preservice teachers generally saw the two as incompatible. For A1, for example:

Algebra and the way it is taught is [sic] currently being debated by educators - are we teaching algebra effectively or are we taking the easy approach to teaching algebra by teaching rules and not understanding? I believe understanding is crucial but in schools we are told that we don't have time to explore algebra and we must teach the rules and move on ... Although it might be easier to just teach the rules in the earlier years it would be more effective and more beneficial in the long term to encourage and facilitate the students to develop an understanding of variable and the relationships between variables in algebra.

A3 points out that “without an understanding of the concept of a variable, the study of algebra is meaningless and simply becomes the learning of rules and the manipulation of meaningless expressions”. A6 echoes these sentiments:

Overall, I think that algebra is best learnt when it is not seen as a set of rules. I think that teachers need to concentrate on the real mathematics involved rather than blind manipulations. The basis of this is giving proper meaning to variables and a focus on why different things are able to be done.

The preservice teachers had clearly been exposed to alternative methods for the teaching of algebra, while the students had only their school experience to reflect upon. This experience appears to dominate their perceptions of algebra learning - although the technology-enriched program of this study attempted to engage the participants in learning which was meaningful and context-bound, they remain locked in a view of algebra as a collection of rules and formulas, where success is a direct result of memorisation and rote learning.

Aside from their different interpretations of “understanding”, the two groups (students and preservice teachers) appear to share many common perceptions regarding effective algebra learning. A2 points out that algebra learning “is best done by having as many practical examples and activities as possible, followed by theory and questions. It must be done in a logical and sequential way or pupils will be lost and not understand what is going on”. A5 states clearly, “I think students best learn by gradually doing each section. Some students need to learn the rules by rote, others learn by using different methods”. She goes on to point out the importance of repetition, sequencing and “being shown”:

I still think a lot of students learn algebra by learning the rule by rote. They don't fully understand the reasons for what they do. Personally I think the students would learn better by being given a thorough explanation and many worked examples. The students then need to do a lot of practice of what they have learnt. They should learn one section at a time, so they don't get confused. Then as they progress they will be able to add all the skills and rules they have learnt to solve more complicated problems.

Although the preservice teachers recommend concrete methods and number patterns for developing algebraic understanding, they appear in general to reflect traditional values associated with algebra learning - that success follows from memorisation and repeated exercises. Their view of learning appears to be strongly teacher-centred, reflecting the view of the students. Only one of the preservice teachers (A3) mentions group work, and that is in a limited way (“The ability to work in group or paired exercises would be advantageous in the early stages of algebra”). Their understanding of the algebra learning process suggests a hierarchy of skills, built upon numerical foundations (echoing Patrick's comment) and requiring the ability to interpret symbols.

A1 defines a “successful lesson” as follows:

A lesson is successful when the students understand the content of the lesson and are able to do the exercises without great difficulty. However, it is hard to pinpoint what made the lesson successful. I think it basically comes down to a number of uncontrollable factors:

- the way the teacher explains the examples and the use of the formula/rule, sometimes the students understand straightaway what the teacher is talking about and other times what the teacher says goes right over the students' heads.
- time of day - morning lessons are always more successful than afternoon lessons and Monday's lessons are more successful than Friday's lessons.
- the content area - is it interesting to the students, is it relevant to them?

Even within an increasingly student-centred context, the successful algebra lesson revolves around the activities of the teacher (A2):

The things that make a lesson successful are good easy to follow explanations, followed up by a student explanation to see if they are following everything that is going on. The examples used in demonstrating the examples [sic] are relevant to the pupils' level and reinforce the explanation. The examples would show step-by-step the way the concept works and would be easy to follow. The students could do these up on the board, as the students often enjoy this. A quick quiz would show to the teacher if the pupils understand what it is they are meant to be doing. A discussion on the topic would show what misconceptions the pupils have and a game on the topic makes the pupils more interested as they think they are not “doing maths”.

The differences between the two groups appear minimal in comparison with their similarities. Both groups make specific mention of teacher-centred methods which involve demonstration, repetition and carefully sequenced exercises leading to memorisation of skills of manipulation, generally associated with success within this domain. The common views of the nature of algebra and the ways in which it may best be learned span both groups of participants, and appear to suggest the existence of what might be termed a **mathematics learning culture** (where the term “culture” is used in the sense of shared meaning and experience). The existence of such a phenomenon has significant implications for the use of computer technology if it is as pervasive and uniform as it appears. Further evidence as to the existence and nature

of such a culture may be gained by triangulating the various sources of data related to algebra learning which this study makes available, particularly responses to the Constructivist Learning Environment Scale and the “grand tour” question, “Describe a typical mathematics lesson”.

A Typical Mathematics Lesson

When asked to describe “a typical mathematics lesson”, the responses of the various student participants were almost identical - after a momentary pause, each began to write, confident at last that they were being asked about something in which *they* were the experts, about which they could respond confidently from experience. The various responses are quoted below, providing as they do valuable insights into the learning experience which all appeared to share and which, more than any other group of data in this study, offers convincing evidence for the existence of a pervasive culture of mathematics learning.

My own response was among the most extensive and suggestive of a relational level of thinking, as opposed to the largely multistructural responses of the younger participants.

A typical maths lesson begins with work set by the teacher - perhaps a few quick review questions, or review of the previous night's homework. This 'housekeeping' is an important part of beginning the lesson, preparing students for work.

The body of the lesson typically involves the teacher demonstrating some new concept or technique, and then (after questions from the students) having them practise this - usually using questions from a textbook. The teacher will usually move around the class while this deskwork is in progress, assisting, answering questions and also keeping students on task.

Occasionally, the teacher will call attention back to the front to explain further some point that may need clarifying, or to set some new work.

In the last few minutes of the lesson, opportunities are again provided for questions, homework is set, and students continue with the work or begin the homework.

For Stephen, the components of the typical lesson are distinct and unrelated parts which occur within a specified sequence:

A typical maths lesson begins with any problems from homework to be answered and explained. Then the teacher will show us the new work and explain it. Then we will be shown a few examples. We will go on with the new work and if we need any help the teacher will show us what to do.

This same sequential element is present in Jane's description, although she makes more mention of physical spatial components:

The teacher begins by writing questions on the board - sometimes general questions, sometimes related to what we are doing. We answer the questions, correct them, then we might work on questions out of the textbook or we might answer questions from the homework. If work is set then the teacher moves around the class; if not, then she stays out the front. At the end we are set homework for the night.

Ben responded twice to this item, demonstrating the sequential multistructural format already seen, and later a more relational level of thinking about this subject, offering justification for the various components of his typical lesson.

A student is taught set mathematics from a teacher. It begins with a couple of warmup questions followed by work we have been doing on a set topic, half of which is taught and the other half comes from a text book. Then the rest of the lesson is spent working from our textbook. Then we are given some homework.

It starts with a warmup question based on the topic we are currently studying, then we continue on our selected topic learning what we need to know, then we are given homework to help us develop our new skills. The teacher explains the reasons behind the mathematics involved and helps us to learn the techniques involved. The work comes out of a textbook.

Although generally more detailed than the student responses, the replies of the preservice teachers to this question might have been drawn from the same classroom. A5 offers a succinct account which effectively captures the recurrent elements of the previous responses:

A typical Maths lesson is one in which a new concept is taught at the beginning of the lesson, then it is practised by the students throughout the lesson. The practice consists of examples from the textbook or a worksheet.

A6 offers a more detailed description, but one easily recognisable as containing the same themes:

From my experience, a typical maths lesson begins with the teacher explaining new work or revising previous work. The students generally have the opportunity to ask questions during this process. After the explanations the students are usually asked to complete a set of exercises. These exercises typically come from a text book and involve drill and practice style learning. The students will usually work individually although some communication is often allowed. The teaching is almost always 'chalk and talk' with the emphasis being on instruction rather than discovery.

Eleven major descriptors may be identified from the responses to this aspect of the study, and these are summarised and displayed with their respective respondents in Table 7.1.

Table 7.1

Descriptors for a Typical Mathematics Lesson

	A1	A2	A3	A4	A5	A6	Ben	Jane	Stephen	SMA
Teacher										
Explanation										
Tchr.										
Demonstration										
Review / Examples										
Homework										
Textbook										
Teacher Questions										
Student Questions										
Student Deskwork										
Teacher at front										
Teacher moving										
Discipline ("on task")										

There can be no more convincing argument for the existence of a mathematics learning culture than that presented in these vignettes. Although some are more detailed than others, all could have been descriptions of the same mathematics classroom, taught by the same mathematics teacher. The power of such a culture to influence the participants on both sides of the desk - teachers as well as students - must be recognised if the impact of any innovation is to be understood. This is a culture which is dominated by the influence of the teacher - each and every description begins with a reference - either direct or implied - to the actions of the teacher. The active role of the teacher is strongly contrasted with the passive role of the students, who are required to work through set exercises and examples - most commonly from a text book - in order to learn how to answer the “problems” and “questions” which appear to define the subject area.

A view of mathematics as answer-oriented is significant within the context of computer tools which appear to be of most use within an open-ended exploratory role - a role which appears to have little or no value within a culture dominated by the finding of a predefined “answer” through a specified sequence of steps involving some manipulation of rules and formulas. This pedagogic role of technology is explored in greater detail in the following chapter; the nature of the mathematics learning culture is examined more closely now through consideration of images and metaphors associated with more and less effective teaching.

Pedagogical Images and Attitudes

This research activity involved participants in the verbal description of a series of “teacher roles”, followed by their categorisation into those associated with more and less effective teaching. Associated as it was specifically with teaching, not all participants were required to complete this task; all Group A preservice teachers provided responses, as did the students Ben and Tony, and myself, as researcher. Table 7.2 summarises those roles considered to be associated with successful teaching () and those associated with unsuccessful teaching (x) for each respondent.

Table 7.2

Successful and Unsuccessful Teacher Roles

Teacher as...	SMA	Ben	Tony	A1	A2	A3	A4	A5	A6
Entertainer									
Police Officer	x	x	x		x	x			x
Gardener									
Captain of the Ship						x			
Travel Agent								x	
Social Secretary	x		x	x	x		x	x	
Tour Guide			x					x	x
Administrator		x		x		x		x	
'The Boss'			x	x	x	x	x		x
Other?		<i>student</i>	<i>Diving instructor</i>		<i>Counselor</i>		<i>Politician</i>		

Responses to this task were varied, some participants recognising that certain roles may have both positive and negative associations with

regard to teaching. Generally, however, those roles most associated with discipline and authority (Police Officer, Captain of the Ship and 'The Boss') were perceived as less successful than those which involved making the learning interesting (Entertainer and Travel Agent) and guiding rather than forcing the learners (Gardener).

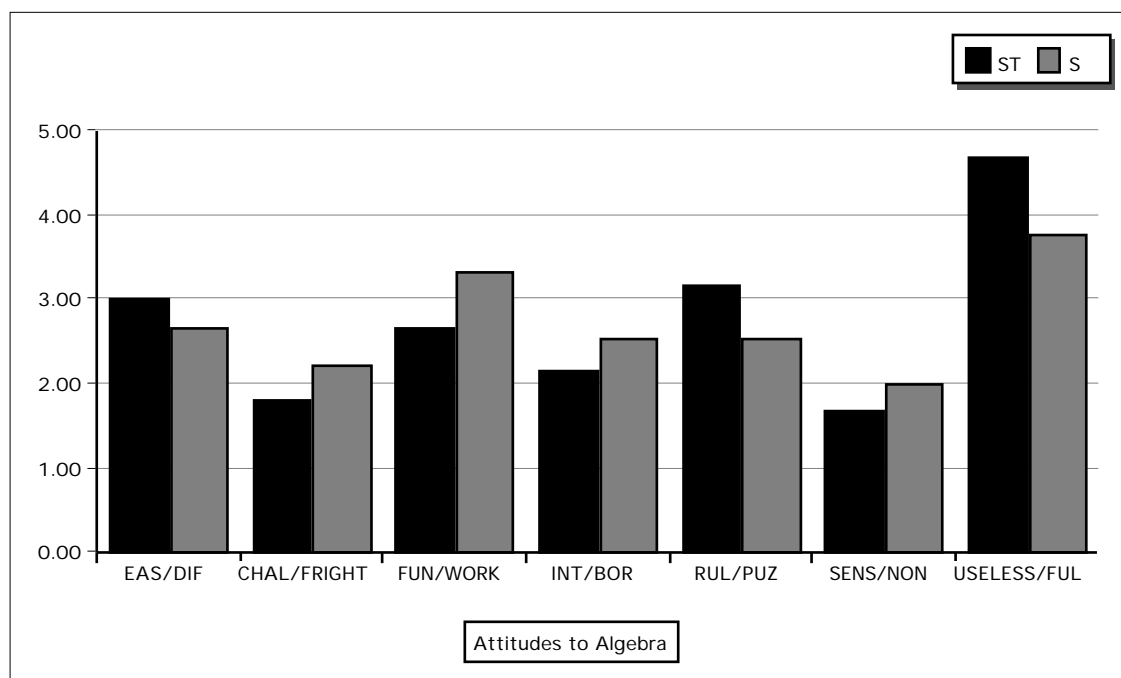
Participants' own categories also proved revealing of their perceptions of teaching and learning. Ben's "student" role ("no-one knows everything and they are still learning") appeared as a negative response to teacher authority, while Tony's "diving instructor" appears relevant in a context of mathematics as a specific set of skills to be learned. For A2, "the counsellor is the person who the pupils can turn to in times of crisis and is one who can offer solutions to their problems. The counsellor shows how to deal with situations effectively". Such a caring view of teaching appears not to be adequately covered by the other roles, and implies a duty of care associated with teaching beyond that of simply imparting knowledge and skills. The "politician" role described by A4 related to the public nature of teaching - "A teacher like a politician must remember that they are always in the public eye and must act accordingly: a teacher is a role model in and out of the class". As with the counsellor, this view of teaching extends beyond pedagogical responsibilities.

Once more we find that the various participants appear to share more common views than differences. The teacher is recognised as the critical factor in mathematics learning and, although some resentment is evident related to abuse of authority ("a teacher... should not be a gestapo-type disciplinarian" (A4) and "teacher abusing authority given

to them” (Ben)), the effective teacher is nonetheless one who is “in charge” and responsible for student learning.

The use of images and metaphors associated with teaching appeared to induce in some respondents emotive responses which touched upon attitudes towards both teaching and learning, and mathematics itself. These were particularly evident in association with issues of discipline and control. Attitudes to algebra learning were recorded using a simple seven item Likert-scale task (Appendix C), and the responses from the two groups were averaged and graphed in Figure 7.1.

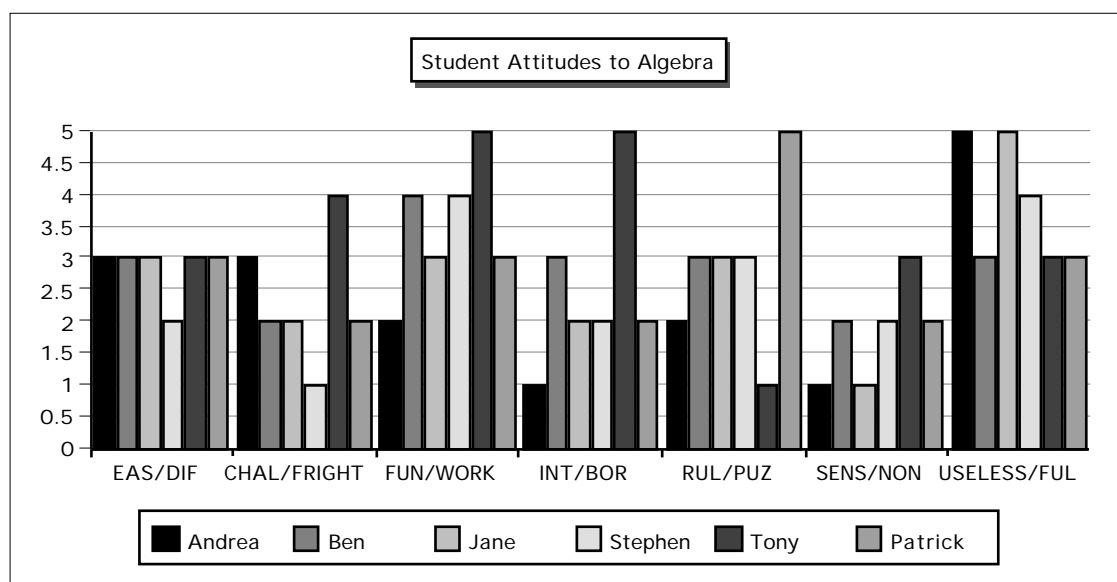
Figure 7.1: Attitudes to Algebra: Comparison of averaged responses from student teachers and students.



Almost all responses indicated positive attitudes towards algebra. Visually, bars below a score of 3.00 may be considered positive. The major exception to this tendency occurred on the last item - both groups considered algebra to be more **useful** than **useless**, with the

preservice teachers being more positive in this regard than the students. This supports the earlier data in which applications of algebra were virtually unknown and its primary purpose is seen as supporting further study of mathematics. Students saw algebra as more **work** than **fun**, while student teachers perceived of it as more like **rules** than **puzzles**. Overall, the students appeared to display more positive attitudes to algebra than the preservice teachers, but with few overall differences between the two groups.

Figure 7.2: Student Attitudes to Algebra



Among the students themselves, there is again general agreement regarding perceptions of algebra as more positive than negative - more “challenging” than “frightening”, more “sensible” than “nonsense”. Most negative of the older students is Ben (S2) who finds algebra more **boring**, less sensible and less **useful** than any of his older peers. Most positive is Andrea, who appears to both enjoy algebra and find it interesting, but at the same time recognises it as both challenging and

frightening. At the time of completing this task, Tony's attitudes towards mathematics were quite negative, although there was evidence of a more positive swing over the period of the study. Patrick, not surprisingly, saw algebra strongly in terms of **puzzles** and more likely to be **nonsense** than **sensible**.

These attitudinal responses help to place the various perceptions of both mathematical and pedagogical aspects of algebra within context for the different participants. They further reinforce the detailed profiles being constructed regarding elements of thinking and action which may influence their use of available computer tools.

Constructivism in Algebra Learning

The most detailed profile data available concerning pedagogical thinking by the respondents was that derived from the version of the Constructivist Learning Environment Scale (CLES) adapted for use in this study. Participants were asked to choose those responses which they would most *prefer* when learning algebra. The twenty-eight items of the scale represented four major groupings characterised by the authors as representing significant aspects of constructivist thinking and practise - **autonomy, negotiation, prior knowledge** and **student-centredness** (Appendix B provides a complete listing of these items). Some aspects of these factors have already been identified with regard to the student and preservice teacher participants. The individual responses to the items of this scale offer a further valuable triangulation method by which both validity and reliability of the associated data may be better judged.

Table 7.3

Responses of Group A student teachers, students and researcher to the Constructivist Learning Environment Scale

	A1	A2	A3	A4	A5	A6	Andrea	Ben	Jane	Stephen	Tony	Patrick	SMA
N1	+	+	+		+			+			+		+
(N5)	+	+	+		+		—	+	—	+	+	+	+
(N9)	+	+	+	+		+	+	+	+	+	+	+	+
N13	+	+	+	+		+	+	+	+	+	+	+	+
N17		+	+	+	—	+		+			+	+	+
N21		+	+	+	—	+		+			+	+	+
(N25)	+	+	+	+		+	+	+	+	+	+	—	+
PK2		+	+		+		+			+	+	+	+
PK6	+	+	+	—	+		+	+		+	+	+	+
(PK10)	+	+	+	+	+	+	+	+	+	+	+	+	+
PK14	+		+	+	+	+	+	—		+	+		+
PK18	+	+		+	+	+	+	+	+	+	+	+	+
(PK22)	+	+		+	+	+		—	+		+		+
(PK26)	+	+	+	+	+	+		+	+	+	+	+	+
A3		+	+	+		—			+	+	+	+	+
A7	—			+			+		—	+	—	+	+
A11				+				—	—			+	+
A15	—		+	+	+	+		+	+		+	+	
A19			+	+	—		+		+	+	—	+	+
A23				+	—			+		+		+	+
A27	—			+			—			—	+	+	+
(SC4)		—	—				—		+	—	—		
(SC8)	+	—			+	—	—	+	+				
(SC12)	—			—	+					+	—		
(SC16)	+	—	—			—	—		—		—		—
(SC20)			—	—		—	—	—	—	—	—		
(SC24)		—	—			+	—	+		—	—	—	—
(SC28)				—	—		—	—	—	—	—		

KEY: Item numbers are prefixed with the scale type (N: Negotiation, PK: Prior Knowledge, A: Autonomy and SC: Student centred). ++ indicates a strong positive response, + a positive response, — denotes a negative response, — — a strong negative response and neutral responses are blank.

NEGATIVE items from the scale are parenthesised; responses to these items were reversed for purposes of analysis and presentation.

Table 7.3 provides a visual display of participant responses to the CLES items which reveals an interesting pattern of distribution. All items associated with the categories of **negotiation** and **prior knowledge** drew almost entirely positive responses; items associated with **autonomy** were largely neutral, and the **student-centred** items drew largely negative responses from all participants. This is supportive of the findings already discussed relating to pedagogical thinking by the respondents: algebra learning is perceived to be (and in this case, *preferred* to be) a **teacher-dominated** activity, in which student **autonomy** is not valued highly as a factor associated with success; in general, while most participants prefer to learn socially (the majority of **negotiation** items reflect this factor), **prior knowledge** is also considered an important feature, associated with rote learning and memorisation of rules and formulas already noted. Once more, strong evidence exists for a high level of uniformity across all respondents, suggestive of the common culture of mathematics learning which has been proposed.

Those items which drew the most uniform responses are italicised in the table. One item (PK10) drew the same strongly positive response from all participants: *When I do algebra, I would prefer there to be NOT enough time to really think.* The negative items (such as this one) had their responses reversed in order that high scores be associated with positive responses and low scores with negative responses. In this case, every respondent chose the same option - never.

Other items which drew almost entirely positive responses were:

N9: *When I do algebra, I would prefer NOT to be aware of other students' ideas* (Negative item - score reversed).

N13: When I do algebra, I would prefer to talk with other students about the most sensible way of solving a problem.

PK26: When I do algebra, I would prefer that the things I learn about are NOT really interesting.

Each of these strong responses related to a preference by the majority of participants for social interaction as a part of the learning process and a natural desire to be interested and not to be rushed and confused in learning.

Items which drew the most strongly non-constructivist responses from most participants were all classified as measuring “student-centredness”:

SC4: When I do algebra, I would prefer that the teacher gives me problems to investigate.

SC16: When I do algebra, I would prefer that the teacher expects me to remember things I learned in past lessons.

SC20: When I do algebra, I would prefer to learn the teacher’s method for doing investigations.

SC28: When I do algebra, I would prefer that the teacher shows me the correct method for solving problems.

These items paint a clear picture which reinforces the perception of algebra learning already encountered throughout the data. Both students and preservice teachers indicate a preference for an algebra learning environment in which the teacher is the source of both content and method, in which there is a “correct method for solving problems” and that this is the teacher’s method. This is a learning environment in which the responsibility for learning rests squarely upon the shoulders

of the teacher, and in which students are passive participants, waiting to be told and shown what to do.

Differences between the various individuals are not great. Of the students, Ben, Tony and Patrick emerge as rating **negotiation** very highly, with Andrea, Jane and, to a lesser extent, Stephen suggesting a tendency to rely less upon their peers. At the same time, Andrea and Tony indicate the strongest tendency towards teacher-dependence from their responses to the **student-centred** items. Only three respondents (A4, Patrick and myself) indicated a strong tendency towards **autonomy**; most others gave neutral responses to these items.

On item A19 (*When I do algebra, I would prefer that I decide if my solutions make sense*), most students responded in a mildly positive way, while the preservice teachers gave a largely negative response. Their greater mathematical experience had, perhaps, reinforced for them that their judgements are not to be trusted, allowing little room for confidence in their answers. Once again, the students' responsibility for learning is abdicated in favour of external sources of authority - presumably teacher and textbook.

The picture we have painted of the current state of algebra learning is not an optimistic one in terms of constructivist principles. There is clear and diverse evidence for the existence of a culture of mathematics learning within this domain which appears to exist across all participants in this study. It is a culture which values rote learning and memorisation of rules and formulas, where understanding is largely instrumental and in which students prefer to be passive recipients of knowledge and procedures transmitted through teacher and textbook.

There appears to be little motivation for independent study or exploration and students appear unlikely to value or even to trust their own answers to the external “problems” and “questions” by which this domain is defined.

An Overview of Pedagogical Thinking

The learning of algebra, for both students and preservice teachers in this study, is a teacher-dominated activity. It is most often associated with memorisation of rules and formulas through repetition and, subsequently, evaluated by reference to instrumental rather than relational measures of understanding. Mathematics (and algebra in particular) is associated with a fixed body of knowledge and an associated collection of actions and sequential processes which are used to arrive at a predetermined solution through a well-defined series of steps. While algebra is strongly associated with finding answers to problems, these problems are unlikely to relate to non-mathematical areas or real-life concerns; they are largely provided by teacher and textbook. Instruction in algebra is linear and sequential, as students seek to replicate and automate the skills and procedures by which the subject is most readily recognised.

Within this rigid framework, individual differences were observed in relation to a number of variables. Perceived responsibility for learning was associated with active and passive conceptions of the learning of algebra, with students such as Stephen and Andrea displaying a preference for “working out” solutions, while Ben and Jane preferred to “be shown”. Across all participants, however, the onus of responsibility was seen to lie with the teacher, who is expected to “show” the students

“how to do it” and to manage critical learning variables such as sequencing and pace. While teachers might be expected to try to make the learning experience relevant and interesting, their primary responsibility was to lead and enforce learning, even though this was sometimes associated with abuse of authority. Attitudes to algebra varied widely for the student participants, with Ben and Tony associated with the most negative attitudes, and Andrea displaying the most consistently positive responses. Stephen was the only student who rated algebra as more easy than difficult; all other responses were neutral in this regard, suggesting limited confidence in their own abilities.

With regard to constructivist practices and beliefs, the preservice teachers were more strongly positive than the students in relation to both negotiation and prior knowledge scales. Ben and Tony again stand out, rating these factors more highly than their student peers. Least student-centred of the students were Andrea and Tony, although overall responses suggested a preference for teacher-dominance. Patrick’s response to the autonomy items (positive) and the student-centred items (neutral) distinguished him from his peers: it seems probable that his limited experience of formal algebra learning situations had failed to initiate him into the dominant culture: he expected that *he* should be responsible for his own learning, rather than the teacher. In other respects, responses were strikingly uniform, and such a powerful culture must be a clear influence upon perceptions of the role of technology within algebra learning situations.