

Four The Research Design

Classroom life, in my judgement, is too complex an affair to be viewed or talked about from any single perspective. Accordingly, as we try to grasp the meaning of what school is like for students and teachers we must not hesitate to use all the ways of knowing at our disposal. This means we must read, and look, and listen, and count things, and talk to people, and even muse introspectively over the memories of our own childhood.

[Jackson, 1968 in Howe, 1988, pp. 11-12]

This study examines mathematical software use within the context of:

- characteristics of the user, in terms of both algebraic and pedagogical thinking (defined below), and
- characteristics of the learning environment, which includes the software tools themselves and the conditions under which they are used.

Within an action research framework, the study examines ways in which individuals learning about algebra make use of available mathematical software, and embodies the major findings within the ongoing development of a computer-based learning environment and accompanying mathematical tools. A theory of mathematical software use is developed using a grounded theory approach, providing a framework for future use of open-ended mathematical software tools in mathematics learning situations, and identifying features associated with both the potential and the pitfalls of the use of such tools. As explained by Strauss and Corbin (1990),

Formulating theoretical interpretations of data grounded in reality provides a powerful means both for understanding the world 'out there' and for developing action strategies that will allow for some measure of control over it. (p. 9)

The design of the study and the subsequent gathering of data are driven by the following research questions:

- What do individuals (researcher, students and preservice teachers) understand by algebra and its components (especially functions, variables, equations, graphs and tables of values) and how might such understandings be related to the use of computer tools?
- What do individuals perceive when they view algebraic objects and how may these perceptions influence their choice and use of available strategies (including the use of mathematical software)?
- What beliefs do individuals bring with them to algebra learning situations concerning the nature of algebra, the ways in which it may best be learned, and the characteristics of successful learning and effective teaching practice? To what extent may such beliefs impact upon the use of technology as a learning strategy?
- Under what conditions do individuals choose to use available software tools, and what forms does this use take? What features of both tool and learning situation serve either to impede or encourage such use?

The Research Instrument

As a means of generating and gathering data related to the focus questions defined in Chapter One, a *HyperCard*-based research tool was developed: *Exploring Algebra* (Arnold, 1993), when used with appropriate advanced mathematical software, provides a tool by which research questions regarding the issues described above may be addressed. By providing an immediate record of user actions as they engage in the use of available tools, and making explicit aspects of their thinking and understanding arising from such use, it potentially offers a powerful means for collecting data on both pedagogical and mathematical thinking at all levels. When used in conjunction with a range of pre- and post-use data collection activities, the possibility exists for describing and explaining elements of thinking which are of critical importance in mathematics learning within a tool-based context. The research method is recursive, as the computer assumes dual roles as both object of focus and primary method of inquiry.

In order to create an appropriate research instrument, it was necessary initially to structure an “algebraic learning environment” - a series of instructional modules which attempted to synthesise the major findings of research into algebra learning of the last decade. These would then provide context for the use of additional mathematical software tools. Although the tools under investigation in this study are open-ended (presenting the user essentially with a “blank page” for computation) their use does not occur in isolation. The context of this use becomes critical in seeking to describe and understand the interactions of teachers and students with such tools. By creating and structuring such a context, the research instrument allows this

variable to be controlled in the design; by recording the interactions of the users with computer tools within such contexts, it becomes possible to capture a detailed record of both action and written description of their parts. Additionally, this research and learning context itself becomes a focus for development within the action research framework of the study.

In order to better understand the design of the research tool under discussion, then, it becomes necessary to first describe its nature as an “algebraic learning environment”.

The Computer as an Algebraic Learning Environment

Open-ended mathematical software of the type described here is never used in a vacuum. Rather, its use is dependent upon context and the characteristics of the individual learner. In order to focus upon these individual characteristics, and to control for the contextual element, a series of instructional modules was designed using *HyperCard* on the *Macintosh*, with the following as primary considerations:

- To provide simple and immediate access for students and teachers to the powerful computer tools by which the teaching and learning of algebra across the secondary school might be enhanced (these included computer algebra, graph plotting and tables of values as central tools, supported by *LOGO* and other quality mathematical software which the user might have available).
- To provide meaningful and challenging contexts for the use of such software tools, creating a learning environment which encourages exploration of the key concepts as they arise, and

providing the means by which such investigations might be carried out. The instructional modules developed towards this end were conceived as providing a starting point from which advanced mathematical software might be used to enhance the learning of algebra.

- To provide a means of monitoring the progress of those who would work through the program, supporting the collection of data on the path taken through the modules, the time spent at each stage, the choices made, in addition to any comments made and answers to questions in the materials. The record of transactions with the materials is collected and saved as a text file, which may then be read directly into the chosen qualitative analysis tool, *NUD•IST* (Richards and Richards, 1993).

The instructional basis for the program builds upon the considerable research of the past decade into algebra learning and misconceptions, representations and the development of concepts such as function and variable, so central to success within this domain. Such research in algebra learning appears to be converging to recommend the following changes to existing instructional and curricular patterns (Australian Education Council, 1990, NCTM, 1989, Romberg *et al*, 1993):

- A focus upon “functions” and “variables” as the organising principles for algebra learning in the secondary school, moving away from the previous focus upon “equations”.
- Increased emphasis upon teaching the ideas and manipulations of algebra linked to and embedded in real-life contexts.

- Increased versatility in the modes of representation used to describe these algebraic ideas.
- Concrete foundations for algebraic manipulations using manipulatives (possibly linked to computer representations).
- Early and more extensive focus upon PROCESS (operational) conceptions of algebraic ideas and decreased emphasis upon the OBJECT (or structural) focus which has dominated to the present day.

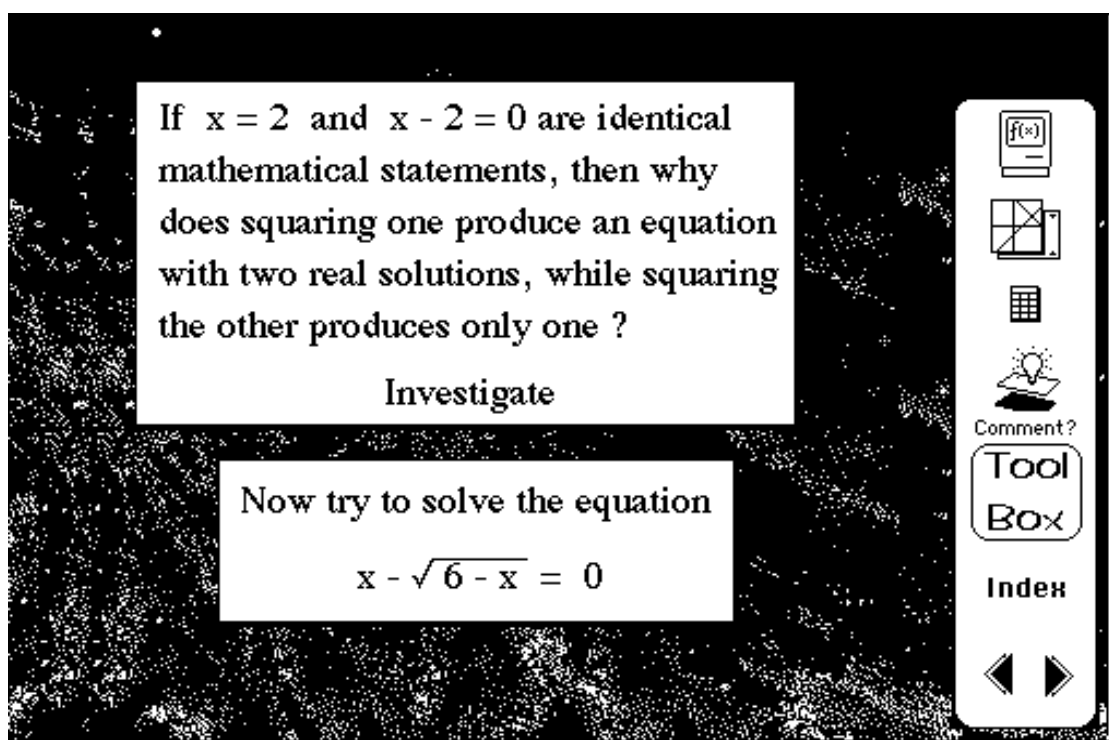
The implications of such research, then, are essentially threefold. As much as possible, the algebra learning environment should provide *context* and *meaning* for the algebraic ideas and processes presented; it should increasingly emphasise the *process* dimension through frequent reference to the numerical and operational bases of the algebraic ideas they encounter, and students must experience a variety of *representations*, developing the skills to move freely among these.

As described above, the program was conceived initially as a means of placing powerful mathematical software at the fingertips of teachers and students in mathematics learning situations. As such, it was considered important to provide two scenarios - an open-ended “workspace”, where expressions of interest might be entered and investigated, supporting a range of instructional modules where examples and background information might be given, questions and problems posed, and the means and motivation provided by which the concepts of algebra may be explored.

Within the instructional modules, each screen (or “card”) provides a “control panel” down the right-hand side, with buttons which will open

computer algebra, graph plotting and table of values tools. Other buttons are designed for navigation (forward and back, return to menu and index cards). A “Tool Box” button takes the user to a card from which any available software tools may be accessed; a “Comments” button allows comments, responses and criticisms to be entered at any time, to be included in the session record.

Figure 4.1: A card from *Exploring Algebra*



In order to further facilitate the use of the software tools, and to encourage exploration, mathematical expressions throughout the program made use of *hypertext* facilities: clicking on any such expression (such as the three equations shown in Figure 4.1) plots the graph; holding down the shift key produces a table of values; holding down the option key opens a selected computer algebra tool, where the expression may be entered simply by “pasting”. In this way, these three

powerful means of investigation and representation are automated and simplified, encouraging students and their teachers to explore the relationships between the various representations.

Exploration is also encouraged through the use of “control key” commands which are available at any time (for example, CTRL-A will open the selected computer algebra tool, CTRL-G will open a graph plotter, CTRL-S the spreadsheet) while the presence of an additional “utilities menu” at the top of each screen provides further access to the tools and other commonly used features of the software.

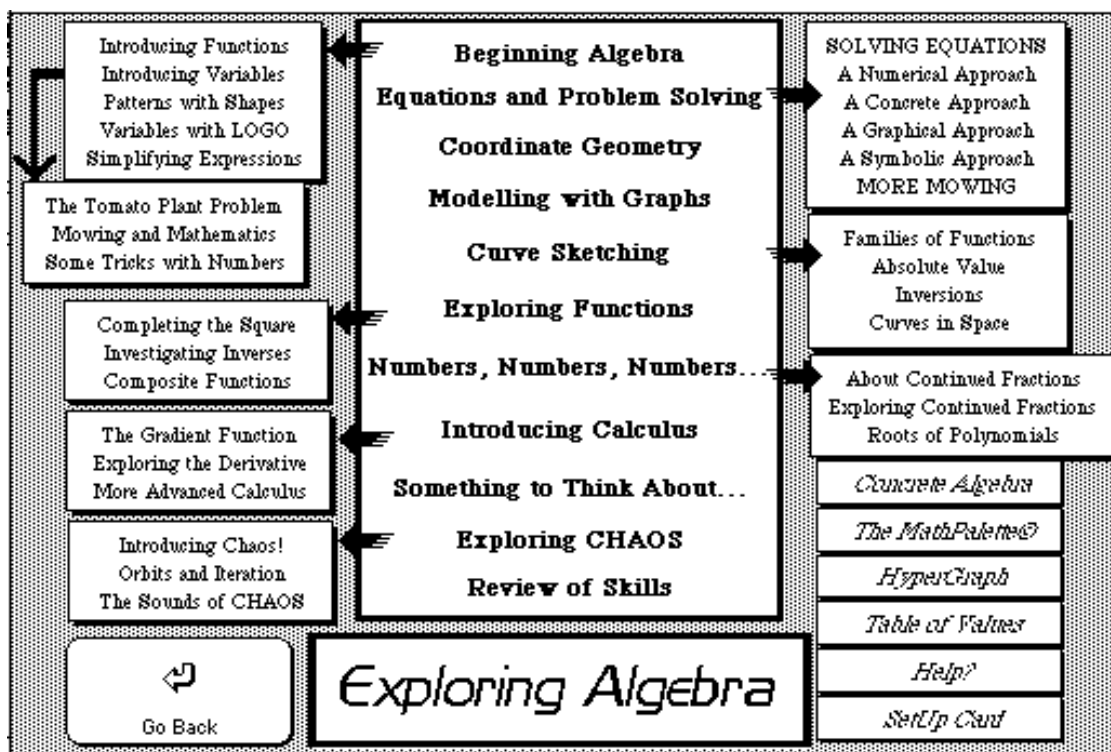
In response to evaluative comments from students and preservice teachers, the “workspace” was subsequently developed to allow the entry of algebraic expressions and equations through a “point and click” interface, removing the need for students to learn the additional syntax normally associated with algebraic and graph plotting software. Initially conceived as simply a tool for simplified entry of algebraic expressions, the development of the *MathPalette* accompanied the gathering of data from students, teachers and student teachers throughout the course of the study. As difficulties were encountered or weaknesses observed in the available software tools, so was the *MathPalette* extended and improved. The many features of this tool - the simple and mathematically correct interface, powerful graph plotting and table of values facilities, the concrete algebra modes, tools for substitution and solving, for coordinate geometry and for “Guess My Rule” games - resulted from interactions and observations over the period of the study, and reflected needs and priorities arising from the data. The research design, then, included a “research and development” component, with the latter an important key to understanding the use

of software tools. The *MathPalette* accompanied the *Exploring Algebra* modules, and was available at any time within the program, encouraging student exploration and mathematical interaction.

The program, then, was designed to encourage the exploration of algebraic ideas in contexts both meaningful and versatile. Students (and their teachers) are provided with tools for developing deeper and richer understandings of the concepts of function, variable and equation, so central to success throughout the study of mathematics. As *LOGO* has been described as a mathematical “microworld”, so was *Exploring Algebra* intended to support a “learning environment” within which deeper and more versatile understandings might be possible.

The Algebraic Context and Research Questions

The mathematical context for the study was provided by the series of instructional modules developed for this purpose using the textual, graphical and interactive capabilities of *HyperCard*. Topics were chosen ranging from introductory algebraic experiences to calculus and open-ended problem-solving. These were then developed in such a way as to encourage the use of available computer tools. The materials were designed to support the development of central mathematical concepts such as function, domain and range, as well as providing context for the learning of algebraic skills, such as equation solving. An overview of the instructional modules is presented in Figure 4.2, followed by a description of each (see Arnold (1993) for a detailed transcription of the content of the modules).

Figure 4.2: Overview of the *Exploring Algebra* modules

Upon choosing a particular module, a *probe* question is asked. Four different probes exist at each section, depending upon whether the individual is a student or “teacher” (referring to preservice teachers) and whether this is the first occasion that the individual has chosen this option or a subsequent choice. The probes centre upon student thinking about the objects of algebra (functions, variables and equations) and pedagogical aspects of the module for preservice teachers. Each time a session commences, additionally, a focus probe requests an explanation of the user’s understanding of algebra, and how it is best learned. Although students were not required to answer this query every time, it provided a central recurring theme in the data from all participants. Importantly, each time an algebra, graphing or other software utility was used, a probe would follow which queried the

nature of this use and the effectiveness perceived by the user. This was a critical component of the research design.

- **Beginning Algebra** provides an extensive introduction to the ideas of function and variable using a variety of approaches. Upon choosing this module, the initial teacher probe was: “As a teacher, how would you sequence an introduction to algebra?” (This initial probe is referred to subsequently as **T1**.)
The subsequent teacher probe was: “What skills and understandings do you consider essential for students to be successful in algebra?” (Henceforth, **T2**.)
The initial student probe was: “What things do you consider essential for success in mathematics?” (**S1**)
The subsequent student probe was: “What things do you consider essential for success in algebra?” (**S2**)

Responses were open-ended, entered from the keyboard at the prompt. The questions were devised to deliberately explore thinking related to understanding of mathematical concepts and perceptions of effectiveness in learning. Occurring at both beginning and end of each major topic, they attempt to capture changes in thinking which may have resulted from the interaction.

The module consisted of five sections, each stressing a different aspect of introductory algebra:

- (i) *Introducing functions*: a textual and graphical development of the function concept using real-world applications (families, boyfriends/girlfriends) to introduce the uniqueness property of function and, at the same time,

the concept of “ordered pair” and its graphical representation. This section concluded with a mathematical application using a “function game”, in which the player is invited to enter numbers and observe a numerical output, and so to guess a selection of simple “number rules”. The user is *prompted* to use a simple table of values representation and, if desired, to observe the corresponding graphical form.

Probes at the conclusion of this section were:

T1: *How would you describe a function?*

T2: *How would you describe a function now?*

S1: *What does function mean to you? Please give some examples.*

S2: *Do you feel that you understand function any better now? How would you describe a function to a friend?*

(ii) *Introducing Variables:* The concept of variable is similarly introduced using real-world applications: first, the “Tomato Problem”, in which students use tabular information to deduce first numerical and then symbolic relationships. This is followed by “Mowing and Mathematics”, an open-ended problem-solving experience in which students are encouraged to use tables of values and graphs to initially describe and then infer advantages and disadvantages arising from the problem situation. Finally a mathematical application - “Some Tricks with Numbers”, in which students play the traditional “Think of a Number” game, supported by tables of values and computer algebra. In this way participants are introduced

to the three major representations and the tools by which these may be accessed.

At the end of this section, participants were again probed:

T1: *How many different types of variable could you describe?*

T2: *What does variable mean to you? Please give some examples.*

S1: *Do you feel that you understand variables any better now? How would you describe a variable to a friend?*

S2: *How would you describe a variable now?*

(iii) *Patterns with Shapes* builds further upon the use of the tabular representation as a means of thinking about situations involving variables. In this case, the derivation of numerical patterns from sequences of geometric shapes (building triangles, squares and other shapes using “matchsticks”) is facilitated by a tabular array. In this form, students are invited to make conjectures and to derive both verbal and symbolic descriptions of the patterns they observe. Probe questions were:

T1: *As a teacher, how do you feel that patterns relate to functions and variables?*

T2: *As a teacher, what value do you see in using patterns to introduce algebra?*

S1: *Have these patterns helped you to understand algebra any better? If so, how?*

S2: *How do patterns relate to algebra?*

(iv) *Variables with LOGO* briefly introduces simple LOGO procedures for building squares, triangles and hexagons

(assuming some limited prior experience) and then moves on to the use of variables as a means of generalising these figures. Students are invited to use *LOGO* and variables to further explore recursion in geometry.

- (v) *Concrete materials* have increasingly assumed a significant role in early algebra, and this final section introduces formal symbolic notation using concrete representations, in which letters are assigned to represent areas of given shapes. In this way, students recognise that a letter so defined has a particular numerical value which is arbitrarily defined by the model, and that such letters may be manipulated and simplified, while retaining their numerical links. This important representation is linked to an interactive *HyperCard* model, by which students are invited to create their own algebraic expressions using shapes provided, to substitute values into these and then to evaluate the results (Figure 4.3).

Final probe questions for the module were:

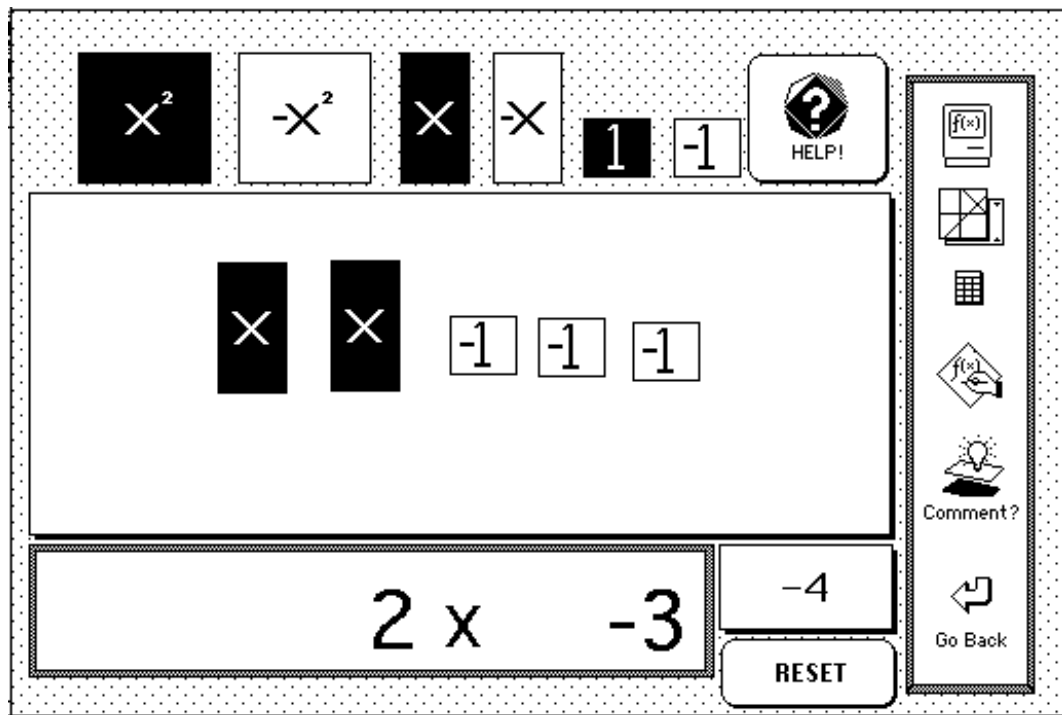
T1: *As a teacher, what value (if any) do you see in using concrete approaches in algebra?*

T2: *How might concrete approaches be used in class by students learning algebra?*

S1: *Do you feel that you understand algebra any better now? How would you explain a variable to a friend?*

S2: *Do you feel that you understand algebra any better now? Please explain.*

Figure 4.3: Concrete Expressions card



The module concludes with a short review section, consisting of ten multiple choice questions related to basic generalisation, simplification and substitution of values.

- **Equations and Problem Solving** continues the versatile introduction to the important ideas of function and variable applied to equation solving. Again, the manipulative aspects of the process are left until last. Emphasis is placed upon numerical, graphical and concrete methods before symbolic approaches are introduced. The module consists again of five sections, the four approaches just described, followed by an application based upon the “Mowing” problem introduced in the previous module.

Figure 4.4: Equations in context

$C = 4d + 7$

In the equation $C = 4d + 7$, there are three terms: C , $4d$ and 7 . Each represents a cost in dollars.

Try and describe in your own words what each term tells us about hiring a taxi from this company.

Discuss your answer with others in your group, and then click on the terms in the equation above to see one way of describing them.

Initial probes were:

- T1:** *As a teacher, how would you sequence the topics in an introduction to equation solving?*
- T2:** *What skills and understandings do you consider essential for students to be successful in solving equations?*
- S1:** *What things do you consider essential for success in solving problems in mathematics?*
- S2:** *What things do you consider essential for success in mathematics?*

Concluding probes were:

- T1:** *As a teacher, how would you approach teaching equation solving now?*
- T2:** *How might concrete approaches be used in class by students learning equation solving?*

S1: *Do you feel that you understand equations any better now?*

How would you describe a variable to a friend?

S2: *Do you feel that you understand equations any better now?*

Please explain.

Again, a ten-question quiz concluded the module.

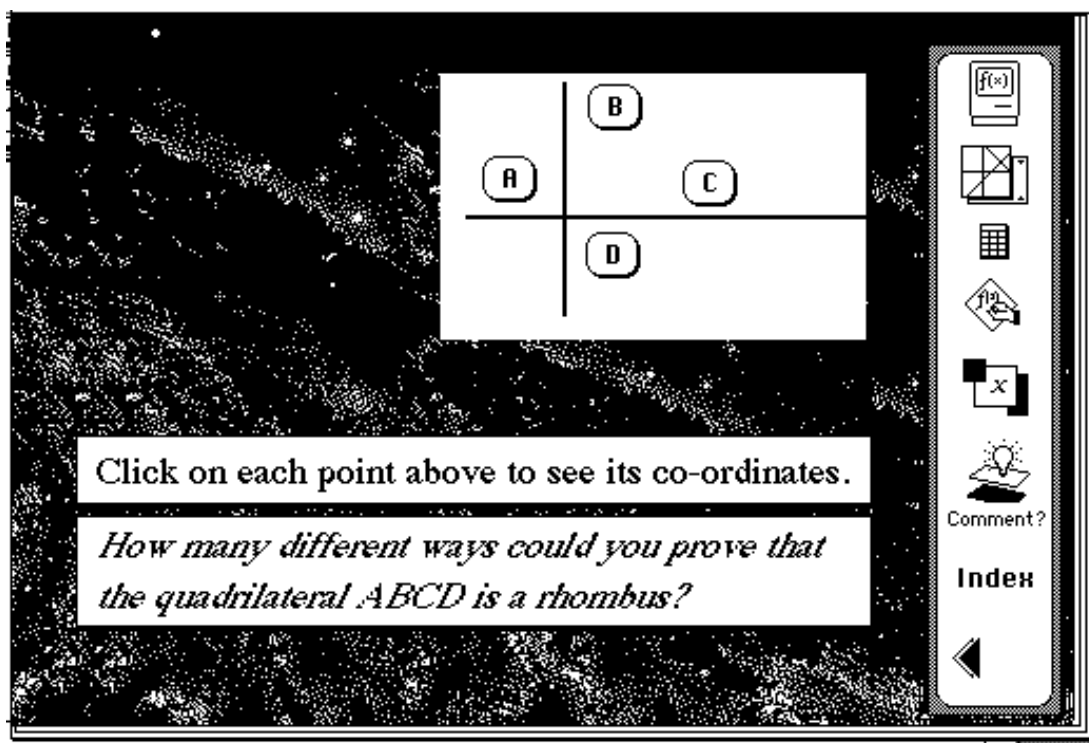
- **Coordinate Geometry** is presented as a series of problems, for which students are invited to use the computer tools available. Again, manipulative aspects are deferred until students have used the various mathematical tools - midpoint, distance, gradient and equation of a line - within various contexts. These mathematical tools are readily available using either the *HyperCard* based graph plotter provided, or the mathematical application, *MathMaster 2.21*, which supports both deriving equations of lines and the solution of simultaneous linear equations and inequalities.

Initial probe questions were:

T1: *As a teacher, how would you sequence the topics in an introduction to coordinate geometry?*

T2: *“What skills and understandings do you consider essential for students to be successful in coordinate geometry?”*

S1/2: *What things do you consider essential for success in number plane work?*

Figure 4.5: A card from *Coordinate Geometry*

- **The Language of Graphs** takes a similar approach, providing a series of problems involving either interpretation of a given graph, or the derivation of a graph to represent a given situation. This unit was seen as an important preparation for later work involving graph interpretation, particularly in the introduction to calculus.

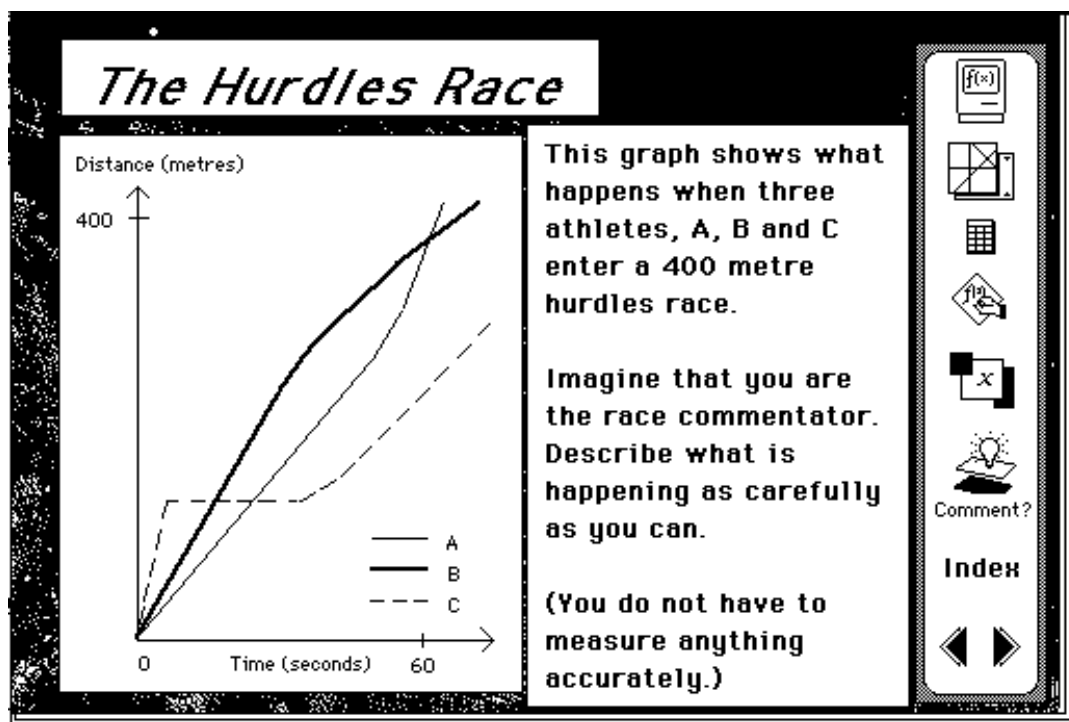
Initial probe questions for the graphs module were:

T1: *As a teacher, how would you sequence the topics in an introduction to graph interpretation?*

T2: *“What skills and understandings do you consider essential for students to be successful in interpreting graphs?”*

S1/S2: *What things do you consider essential for success in understanding graphs?*

Figure 4.6: Modelling with Graphs



- **Curve Sketching** is an extensive unit which introduces families of functions and then examines in detail absolute value and reciprocal functions, as means by which students may become familiar with both the graphical representations of algebraic forms, and the effects of transformations upon both. The module concludes with an extension involving three-dimensional graphs of functions, supported by available software tools (especially *xFunctions 2.2*).

Probes for Curve Sketching followed a similar pattern to those of previous modules:

T1: *As a teacher, how would you sequence the topics in an introduction to curve sketching?*

T2: *What skills and understandings do you consider essential for students to be successful in curve sketching?*

S1: *What things do you think are most important for success in drawing graphs of functions?*

S2: *What things do you think are most important for success in curve sketching?*

Figure 4.7: Curve Sketching

Click on each function to see its graph.

Now compare the graphs of functions such as $y = x - 2$, $y = 2x$ and $y = 3 - 4x$, with each of their reciprocal functions,

$$y = \frac{1}{x - 2} \quad y = \frac{1}{2x} \quad y = \frac{1}{3 - 4x}$$

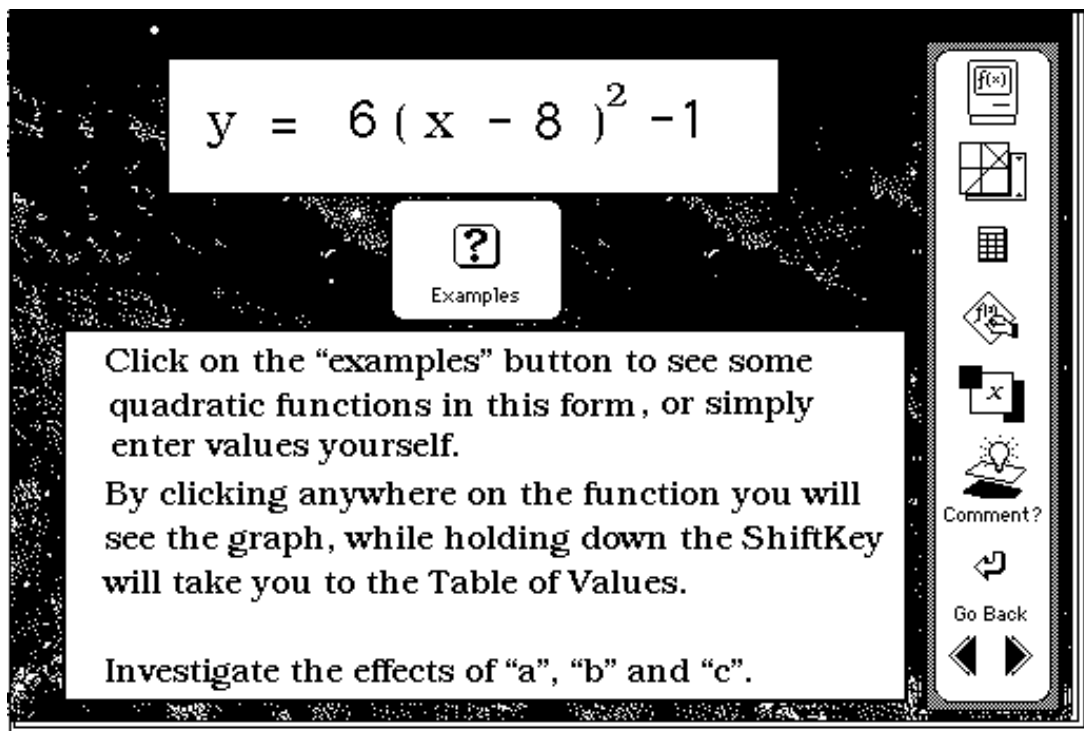
Try now to describe a "rule for inversion" which will allow you to turn any given function "upside down". Compare your rule with others, and try it out on a few more examples.

Navigation icons: $f(x)$, graph, calculator, hand, x , lightbulb, Comment?, Menu, left arrow, right arrow.

- **Completing the Square** is another module which was developed extensively. It explored, not only ways in which computer algebra, graphing and tables of values might be used as tools for exploration, but also interactive aspects of the *HyperCard* programming language which supported its use as a tool for mathematical investigations. Together with the modules **Investigating Inverses** and **Composite Functions**, these provided opportunities for students to explore aspects of

functions which involve higher order thinking - functions acting upon other functions to produce new functions. Each provides text-based information and interactive means of exploring the ideas presented for a variety of functions.

Figure 4.8: An application of *Completing the Square*

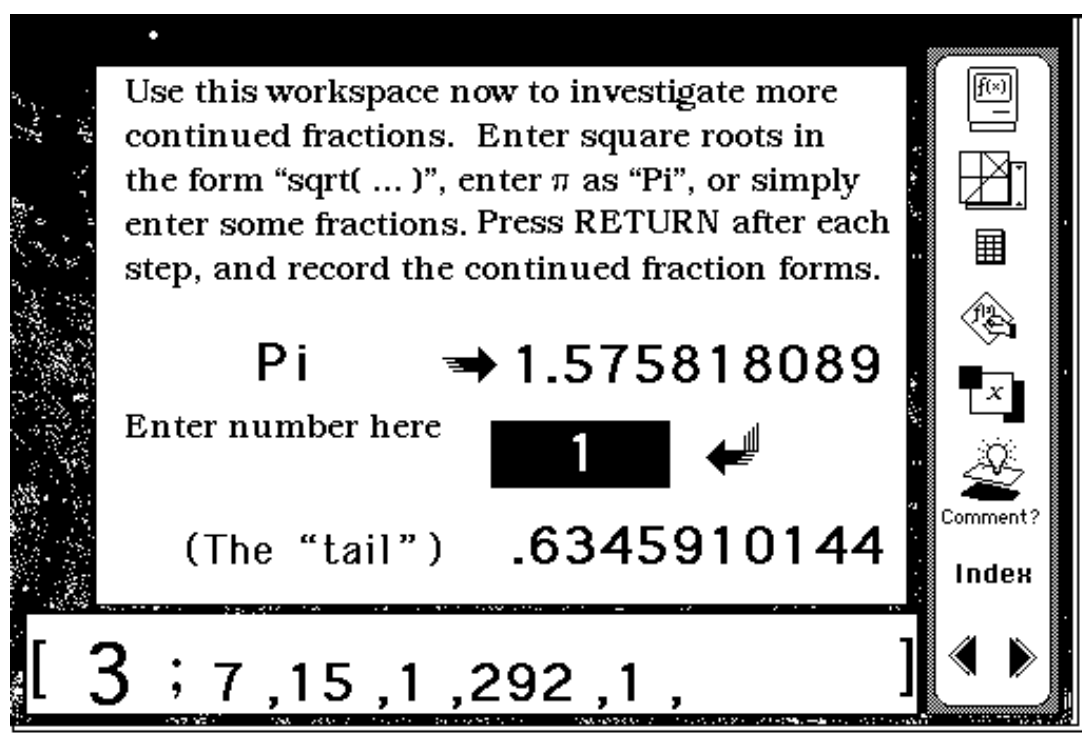


Initial probes for this section (entitled “Exploring Functions”) were:

- T1:** *As a teacher, what do you see as critical to an understanding of functions?*
- T2:** *How would you explain what a function is to a student who did not know? What examples would you give?*
- S1:** *How would you explain what a function is to someone who did not know? What examples would you give?*
- S2:** *What things do you think are most important in understanding functions?*

- **Numbers, Numbers, Numbers...** offered another module which combined related units - in this case, **Continued Fractions** and **Roots of Polynomials**. Both offered means for exploring ideas related to irrational numbers, using both mathematical and computer-based tools.

Figure 4.9: A Continued Fractions Investigation



These units were prefaced by the following probes:

- T1:** *How would you sequence an introduction to the ideas of irrational numbers?*
- T2:** *What skills and understandings do you consider essential for students to be successful in working with numbers?*
- S1:** *What things do you think are most important in understanding irrational numbers?*

S2: *What things do you think are most important for success in working with numbers of all types?*

- **Introduction to Calculus** was designed as a problem-based introduction, in which students could use the graphing and manipulative capabilities of the available software to “discover” the basic rules of calculus. This is in contrast to commonly-used teaching approaches in which the rules are provided by teacher or text and simply learned and practised. The module begins with ideas of “local straightness” (explored using the zooming capabilities of the technology) in order to build an understanding of the critical concept of gradient at a point.

Figure 4.10: *An Introduction to Calculus*

We write $\int_a^b f(x) dx$ to indicate the *integral* of a function $f(x)$ between two points, $x = a$ and $x = b$ *with respect to* x .

EXPLORE and DISCUSS what you think this means using the message box (option-b gives \int and “=” evaluates your expression).
You might use graphs and tables to view a few examples of functions of the form $\int (f(x))$.

∫(2x from 0 to 2)

Again, the initial probes focus upon the teaching sequence for the student teachers and student perceptions of success.

T1: *As a teacher, how would you sequence the topics in an introduction to calculus?*

T2: *What skills and understandings do you consider essential for students to be successful in calculus?*

S1: *What things do you think are most important for success in mathematics at the higher levels?*

S2: *What things do you think are most important for success in calculus?*

This unit makes extensive use of computer support. Students are encouraged to view graphs and tables of values of the functions they encounter as a means of strengthening their familiarity with these forms. Early emphasis in the unit is placed upon the acquisition of skills of *estimation* related to derivatives - students are expected to be able to visually estimate the graph of a gradient function given the graph of the original. They are also provided with graphing and table of values facilities which represent the derivative and integral of a function without giving its symbolic form. In this way they are encouraged to discover the various rules by building upon their knowledge of functions in these representations. Finally, software tools such as *xFunctions* and *CoCoA* are available to be used to provide the symbolic derivative, if desired, allowing students facilities to verify their answers and to further explore the rules they are seeking to establish.

The module concludes with probes related to understanding of functions:

T1: *How would you describe the idea of function now? What examples might you use?*

T2: *How might computer approaches be used in class by students learning about functions?*

S1: *Do you feel that you understand functions any better now? How would you describe an equation to a friend?*

S2: *Do you feel that you understand functions any better now? Please explain.*

- **Something to Think About...** presents a collection of problems related to functions which are intended to provide the impetus for student exploration and real-world grounding and practical applications for many of the ideas and processes they have encountered. Although not all of the problems require the use of computer tools, all are inspired by the technology and encourage careful thought about the central concepts of algebra (especially function, domain and range). Some problems have been chosen to highlight the limitations as well as the advantages of computer software (for example, Figure 4.11 displays an equation for which the graphical representation provides very limited (and indeed, misleading) information, while the table of values displays the solutions immediately).

Initial probes focused again upon understanding of functions:

T1: *As a teacher, what aspects of functions and variables do you consider to be most important?*

T2: *As a teacher, how would you explain the idea of variable? What examples would you give?*

S1: *What things do you think are most important for success in solving problems in mathematics?*

S2: *What things do you think are most important for success in working with functions and variables?*

Figure 4.11: *Something to Think About...*

Find all real values of x that satisfy

$$(x^2 - 5x + 5)(x^2 - 9x + 20) = 1$$

Are you sure you have them all ?

(adapted from NCTM (1988) "The Ideas of Algebra" Reston, VA.: NCTM.)

Toolbar icons: $f(x)$, graphing calculator, standard calculator, hand cursor, x , lightbulb, Comment?, Index, left arrow, right arrow.

- **Exploring CHAOS** was included as an extension module which again provided impetus for mathematical investigation and open-ended problem solving. Attitudes of both students and student teachers towards such material were considered significant in the context of the use of software which demanded such exploration. Initial probes were:

T1: *As a teacher, what value (if any) do you see in introducing topics such as Chaos to students?*

T2: *What do you understand by Chaos Theory? Is it relevant to your students?*

S1: *Do you think that there is any 'new mathematics'?*

S2: *What things about Chaos do you find most interesting? What things do you think are important?*

- **Review of Skills** consisted of six ten-question quizzes which reviewed (in multiple-choice format) basic skills from Year 7 (**Beginning Algebra**), Year 8 (**Equations**), Years 9/10 (**Basic Algebra**), Year 11/12 (**Senior Algebra Review**) and challenge problems (**Stress Test**). The tests were structured in such a way that students could make several attempts, if desired. The first attempt was valued at 2 points, and no computer tools were available. This was intended to simulate the more usual mathematics learning situation, where computer tools are unavailable and students must rely upon their own mastery of algebraic skills. If their attempt was unsuccessful, the tools became available, and the value was reduced by 1. Subsequent attempts were valued at zero, placing some emphasis upon obtaining a correct answer on at least the second attempt. In this way it was anticipated that students might be encouraged to make use of the software tools to at least verify their responses on this second attempt in order to avoid a zero score. Although linked to the first two modules, the review tests could be attempted at any time and in any order.

Initial probes were included for this review:

T1: *As a teacher, what skills and understandings are essential for success in algebra?*

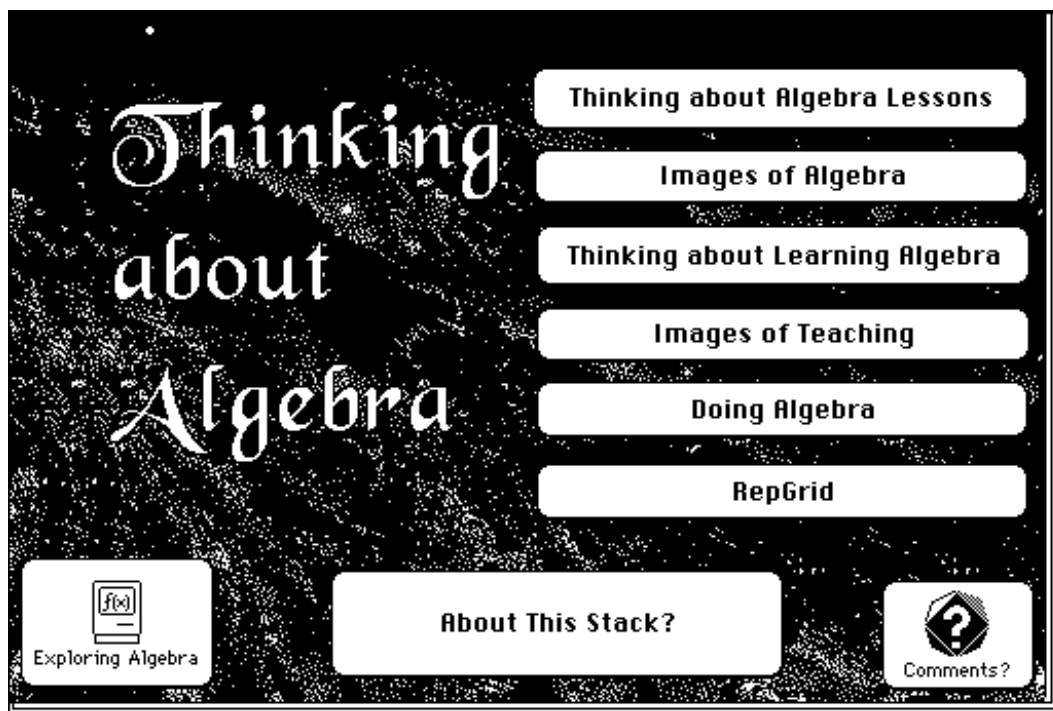
T2: *What skills and understandings do you consider essential for students to be successful in solving equations?*

S1: *What three things do you think are most important for success in mathematics?*

S2: *What three things do you think are most important for success in algebra?*

Supplementary to the algebraic learning context was a series of “research questions”, designed to provide additional data regarding the understandings, attitudes and beliefs of the participants concerning both mathematical and pedagogical aspects of algebra. At the commencement of each session, users were prompted as to whether they had “answered the research questions yet”. They were free to answer “yes” if desired and move directly into the program. If the negative response was chosen, they were presented with a menu of six parts (see Figure 4.12). Not all participants answered all parts; rather these were seen as supplementing the core data derived from the interaction of the individual with the software tools. The research questions were intended to provide a “profile” of each user which might assist in seeking to better understand their responses, and which might provide some degree of comparative data in relation to other participants.

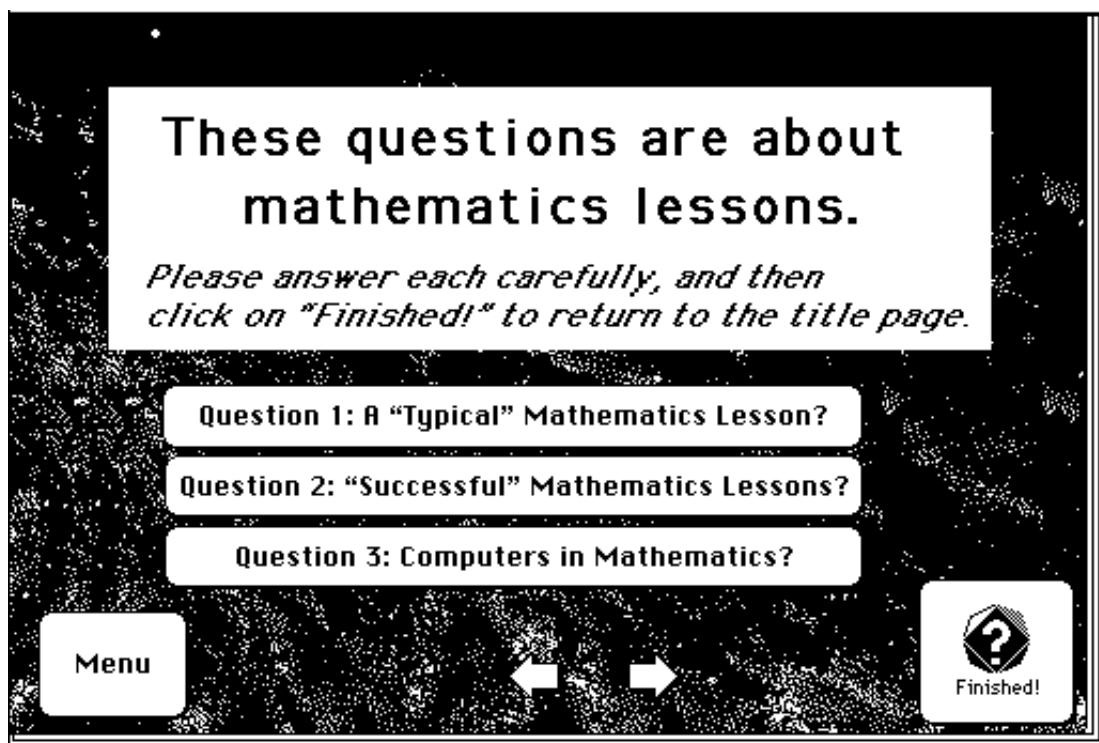
Figure 4.12: The Research Questions



Pedagogical Data about Algebra Learning

Data related to pedagogical aspects of algebra learning was generated specifically by three of the research components. The first, **Thinking about algebra lessons**, consisted of three open-ended questions, which the user was encouraged to answer “as clearly and as completely as possible”. The first asked for a description of “a ‘*typical*’ *mathematics lesson*”; the second asked the participant to “*think back to a time that you experienced a really effective mathematics lesson*”. They were then required to indicate those factors which they believed helped to make the lesson so successful. Finally, the user was requested to describe “*in what ways computers might be used to make mathematics learning more effective*”.

Figure 4.13: Thinking about algebra lessons



Twin central themes recurred with regard to the data collected on pedagogical aspects of algebraic thinking throughout this study, and these are encapsulated in this first section. The notion of a “typical mathematics lesson” was considered important in helping to identify commonalities and differences across the participants with regard to their experiences of mathematics learning. Beliefs regarding the nature of a “successful” or “effective” learning experience were also considered critical in seeking to understand perceptions of the role of computers in the learning process and beliefs regarding algebra learning in general. The recurrence of these twin themes throughout the data collection process was deliberate, seeking to illuminate these concerns from different angles and so to provide some degree of triangulation from which to make judgements concerning the validity of the different responses.

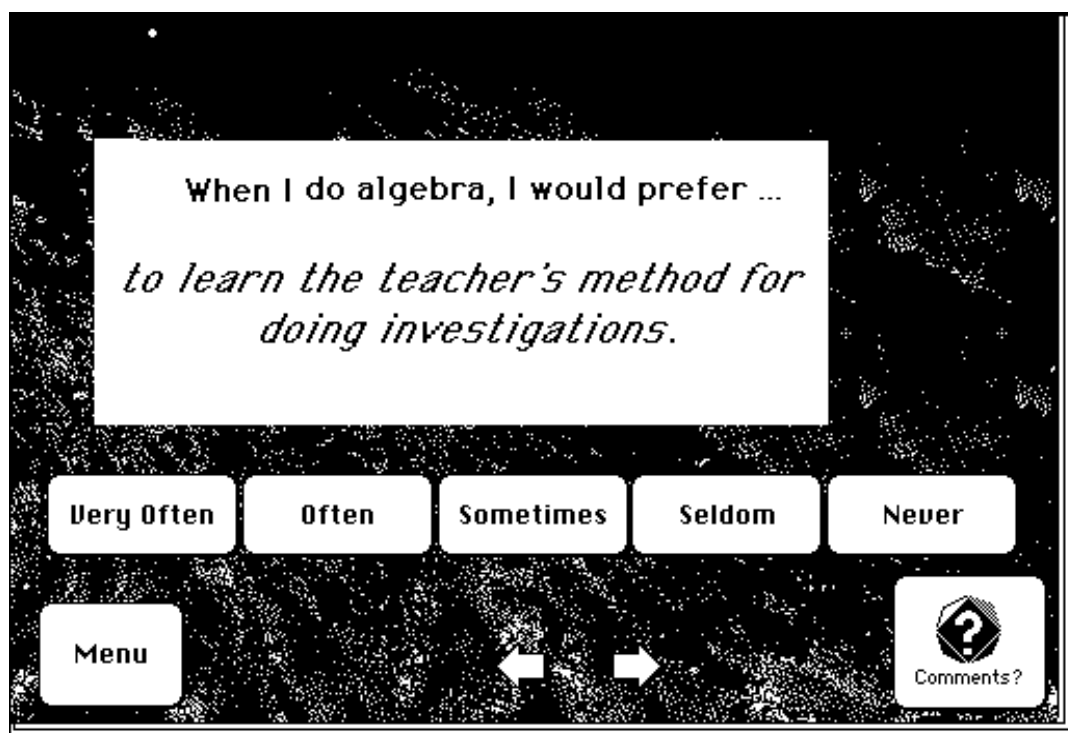
The section entitled **Thinking about learning algebra** consisted of the twenty-eight multiple-choice questions of the *Constructivist Learning Environment Scale (Preferred form) (CLES)* (Taylor and Fraser, 1987). Development of this scale had led to the identification of four factors associated with constructivist learning principles as defined by the authors - *autonomy, negotiation, prior knowledge* and *student-centredness*. The questionnaire adopted for this study consisted of seven questions (both positive and negative) for each of these four factors (see *Appendix B* for a listing of the items). As described by the authors (Taylor and Fraser, 1987),

The **Autonomy** scale measures perceptions of the extent to which there are opportunities for students to exercise meaningful and deliberate control over their learning activities, and think independently of the teacher and other students. The **Prior Knowledge** scale measures perceptions of the extent to which there are opportunities for students meaningfully to integrate their prior knowledge and experiences with their newly constructed knowledge. The **Negotiation** scale measures perceptions of the extent to which there are opportunities for students to interact, negotiate meaning and build consensus. The **Student-Centredness** scale measures perceptions of the extent to which there are opportunities for students to experience learning as a process of creating and resolving personally problematic experiences. (p. 2)

The version of the *CLES* scale adapted for this study consisted of twenty-eight items in a five-point Likert format (*Very Often, Often, Sometimes, Seldom* and *Never*). The unbalanced nature of the scale responses (“very often” should be “always” in order to balance the opposite response, “never”) follows the design as specified by the authors. The absolute positive response was apparently considered too extreme and unlikely to be chosen by students. Although the scale was developed in two forms - *Preferred* (in which subjects indicated responses consistent with their *preferred* mode of learning) and *Perceived* (in which responses related to the way in which they actually perceived their current mathematics learning situation), only the

Preferred version was used. While development of the scale appeared statistically rigorous, it was not intended to be used for statistical analysis in this context, but rather to provide further meaningful data for the participant profile discussed above. It was considered an appropriate instrument for this purpose since it offered quite specific and consistent information regarding participants' preferences and beliefs regarding algebra learning in a format which was well-suited to the computer-based data collection mode adopted.

Figure 4.14: CLES Scale item (Negative Autonomy)



The third research section directly related to pedagogical thinking was called **Images of Teaching**, and consisted of ten cards, each displaying a teaching role or metaphor, beginning with the stem "Teacher as...".

The metaphors initially chosen were: “Teacher as...”

Entertainer

Police Officer

Gardener

Captain of the Ship

Travel Agent

Social Secretary

Tour Guide

Administrator

“The Boss”

?

Space was provided for participants to enter one or more metaphors of their own choosing at the end. Participants were invited to describe what each metaphor meant to them and then, after viewing all cards, to associate each with more or less successful teaching. Research on the potential of metaphors as tools for understanding and improving teaching practice (Tobin, 1990, Ritchie and Russell, 1991) suggests that they might also be effective means of making explicit aspects of the craft knowledge of teaching which is otherwise difficult to articulate. For both students and preservice teachers, perceptions of the role of the teacher (especially with regard to effective lessons) would appear to be critical in the present context and provides another valuable perspective on pedagogical thinking related to algebra learning.

Mathematical Thinking about Algebra Learning

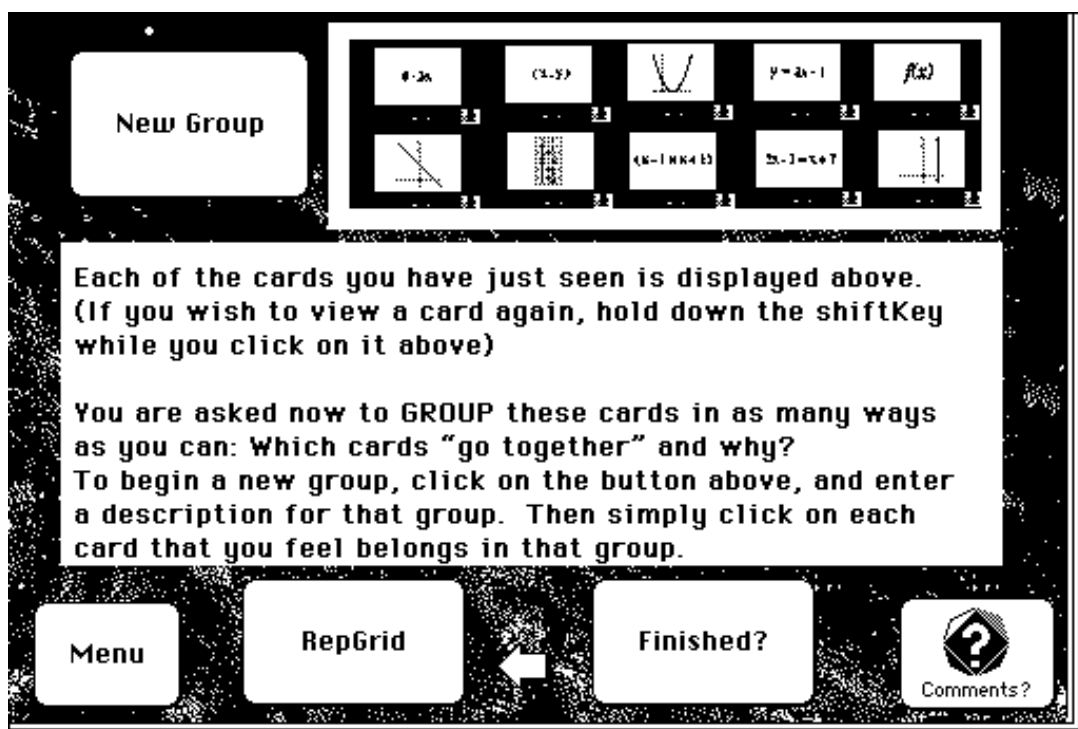
Difficulties associated with articulating tacit knowledge are not restricted to the craft knowledge of teaching. In the present context, they apply equally to thinking about algebra. Getting beyond the

question “What do you think algebra is?” and, further, getting “behind” the answers given to tease out greater detail regarding both the concepts which comprise this complex notion and, more importantly, the relationships between these concepts, proved to be a critical consideration in the research design. The use of cards displaying visual images of algebraic representations as described by Stein, Baxter and Leinhardt (1990) appeared to offer an appropriate technique for eliciting deep aspects of algebraic thinking, especially within a computer-based environment. Ten images were created on cards within *HyperCard* displaying graphs, tables of values, algebraic expressions and equations and symbols related to algebra. Several were deliberately linked (e.g. the expression $(x - 1)(x + 1)$ and a table of values displaying the rule $y = x^2 - 1$). Functions and non-functions were included in both symbolic and graphical forms. Within the ten cards, there was the possibility to go beyond a “surface grouping” of “graphs with graphs”, “equations with equations”, and so on. As with **Images of Teaching, Images of Algebra** invited participants to describe each card in turn, saying what each meant to them and then, after viewing them all, to *group* them in as many different ways as they could. This grouping was achieved on the computer by having the user enter a “new group” name, and then simply click on the small image of each card which belongs in that group (see Figure 4.15).

The use of visual imagery as a prompt to more detailed and rich description of complex concepts such as “algebra” appears to offer a powerful means for accessing and making explicit aspects of individual thinking. Participants at all levels asked to describe “What is algebra?” or “What does algebra mean to you?” were generally found to be extremely limited in their responses; putting complex concepts into

words appears to impose quite significant demands - demands which appeared to be lessened when responses were visual and tactile, rather than linguistic.

Figure 4.15: Grouping Images of Algebra

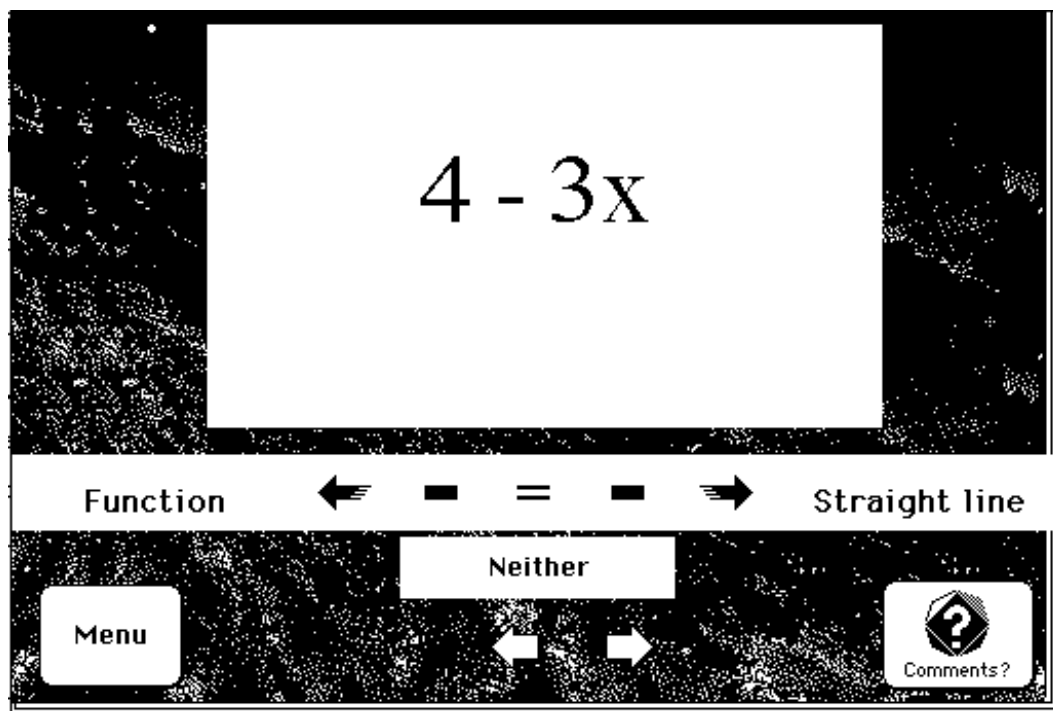


As a means of probing even further into student understanding of algebra, the **Images of Algebra** section was (for some participants) extended and enhanced using a *Repertory Grid* approach, or *RepGrid*. Based upon Kelly's (1955) Personal Construct Theory, the Repertory Grid was developed as a technique for eliciting, not just the components of individuals' thinking about complex concepts, but aspects of the relationships between these components. It has been especially popular as a tool for investigating teacher and student thinking in educational research, offering an attractive blend of data which is both detailed and idiosyncratic in its reflection of individual responses, while at the same

time potentially generalisable and amenable to statistical analysis (Solas, 1993, p. 209).

A common format for *RepGrid* analysis involves deriving a series of statements or prompts related to the particular construct in question (e.g. “good teaching”), then presenting these three at a time (randomly selected) and having the participant describe “In what way are two of these alike, and different to the third”. This forced discrimination generates a new series of constructs which are unique to the individual, usually in the participant’s own words. Finally, these constructs may be applied back to the original prompts, where the participant commonly uses a five-point Likert format to describe the extent to which each of the constructs relates to the original statement. The resulting matrix is amenable to statistical analysis, if desired.

In this study, the ten “images of algebra” were presented three at a time, and participants were asked to indicate by clicking which was perceived to be the “odd one out”, and then give a reason for this choice. For several of the student participants, the reasons were analysed to derive a number of common words or phrases (such as “equation”, “function” or “graph”). These were then applied two at a time to each of the ten images as the extremes of a bipolar continuum. At each card, the participant was asked to decide whether the image shown was more like one or the other, had elements of both, or was like neither. In this way, a very detailed analysis of an individual’s thinking about algebra was possible. Because the process was extremely time-consuming, only a small number of students engaged in this section.

Figure 4.16: *RepGrid* card for *Images of Algebra*

Finally, a simple attitude scale was presented (*Appendix C*). In this way, a measure of both attitude towards algebra was possible for the various participants, both at the commencement of their involvement in the project, and at various points throughout.

The Participants

This study of learning to use new tools begins as a case study of the teacher/researcher's interactions with a single student (labelled below as **S4** and referred to subsequently as *Stephen*) within a tool-rich algebraic learning context. The encounters occurred within individual tutorial situations over a period of almost two years, with some thirty-six hours of interactions recorded and analysed. As the study progressed, it grew to include five other student informants and two

groups of preservice teachers as the cyclic nature of the grounded theory method demanded greater variability within the data, and new research questions and priorities became apparent.

In addition to the teacher/researcher, the study then involves three groups of participants engaged in the use of mathematical software tools. Each provides a unique perspective, intended to illuminate different aspects of the research problem. With regard to the twin poles of mathematical and pedagogical thinking, each group offers a different emphasis. The secondary students, in their dealings with the software tools, are concerned primarily with the mathematical demands of the learning contexts, and only incidentally with pedagogical aspects. The student teachers might be expected to be more interested in the pedagogical elements and implications of the experience, although one group is deliberately influenced to consider mathematical aspects. The sample groups, then, potentially offer the means of comparing and contrasting aspects of software use across different situations. Such a situation allows for considerations of variance within the study, increasing the richness of the theoretical description.

The secondary students

The student group for this study comprised six secondary students who engaged in use of a range of mathematical software within individual tutorial situations over periods ranging from two months to two years. Table 4.1 displays the characteristics of each of the individuals (including a rating of “ability level” from 1 (very high) to 5 (very low) derived from their school gradings and associated with their mathematics course for the senior students). The chosen group may be

broadly viewed as providing a cross-sectional sample across gender, year level and mathematical competence. The principal informant, Stephen, is highlighted.

Table 4.1

The student participants

Code	Name	Sex	Year	Time (hours)	Ability Level
S1	Andrea	Female	11	15	2 (3 Unit)
S2	Ben	Male	12	20	3 (2 Unit)
S3	Jane	Female	10	12	3
S4	Stephen	Male	11-12	36	2 (3 Unit)
S5	Tony	Male	8-9	10	2
S6	Patrick	Male	7	6	2

The research sessions occurred, on average, for one hour per week during the school term. In addition to the gathering of data, the researcher made his services available as tutor for those students who desired help with their associated studies, and this aspect of the professional relationship between researcher and student was significant in influencing the nature of the data collection process. In order to create as realistic a learning situation as possible, this tutoring role was generally allowed to dominate the interactions. Activities solely related to the gathering of research data were minimised, and consequently the majority of data collected was naturalistic. These same considerations served to preclude the use of a tape recorder or video camera as means of gathering data, as such methods were considered too intrusive in what was an essentially private learning

situation. Students needed to feel free to make mistakes and to display uncertainty, and such record-keeping was felt to seriously diminish such freedom.

The use of the computer as a means of recording comments and mathematical interactions proved to be an ideal compromise between the research demands and those associated with the mathematics learning situation. Students indicated a willingness to engage in this form of data collection, in full knowledge that aspects of the learning process were being recorded. Written responses to probes and prompts within the materials, and comments made at various stages in the interactions, were accepted as overt and deliberate records, which students had the option to pass over, or to answer with a degree of care and consideration denied them by other more obtrusive means.

The data collection process involving the secondary students in interactions with the various computer software tools blends elements of clinical interview procedures with participant observation. At all times, the researcher was an integral part of the process, driving and directing both the mathematics learning situation and the research component. This data collection process is considered in greater detail in subsequent sections.

The preservice teachers

Two groups of preservice teachers (hereafter described as Group A and Group B) provided data for the study. Both groups were in the final stages of four-year Bachelor of Education degrees at different institutions. They engaged in the programme as part of their assessment for units of study centred on the role of technology in the teaching of high school algebra. Group A consisted of eighteen students of whom twelve were female; Group B was made up of eight students, of whom only one was male. The strong weighting of females in both groups appears to be most closely related to the nature of the degree programme as a four-year specialist Education degree, in contrast to the other available option - the Diploma in Education, a twelve month post-graduate programme, completed after an initial generalist degree (usually Arts or Science). The latter option allows greater flexibility in career choice than the Bachelor of Education, and the gender balance in the Diploma programme was reported to be more equally distributed. It appears likely that girls were more willing than boys to make a firm commitment to the teaching profession by choosing the specialist degree course; the alternative allowed participants to “keep their options open”.

Group A: The unit studied by Group A required two hours per week class attendance over two semesters (a full academic year). The outcomes of the unit, as specified in the course outline, indicated that students would:

- (i) become familiar with developments in technology appropriate for mathematics teachers.
- (ii) investigate outcome based assessment.
- (iii) become familiar with new changing policies in school mathematics.

- (iv) compare different philosophical viewpoints of knowledge and learning.
- (v) become familiar with topics in the senior New South Wales syllabus.

The coverage of the role of technology in algebra learning, then, was situated within a much broader context. Students were presented with two two-hour demonstrations of available algebra software tools (*Theorist Student Edition*, *MathMaster 2.21* and *xFunctions 2.2*) and were introduced to the *Exploring Algebra* package as a means by which they might investigate the use of the tools within a variety of algebra learning contexts. The remainder of their experience was then dictated by the assessment requirements for the unit.

The project on “*Algebra in Technology*” was one of three major assessment tasks specified for the unit (the other two being a Reflective Journal and a task related to outcomes based assessment), and was allocated 30% of the assessment total for the course. As described in the subject outline (Appendix D), students were required to “work through computer instructional modules and reflect on the teaching and learning of algebra in the context of computer assisted learning”.

Three specialisations were made available, for junior, middle or senior algebra. The majority of students chose the junior option, probably because this was the emphasis of the demonstration of the tools. The assessment weighting favoured the pedagogic aspects of the task, with only 10% of the possible 30% allocated to the open-ended use of the software tools. This had the result that the participants focused upon issues of teaching and learning, and their exploratory use of the tools was minimal. It should be noted, too, that although *Theorist Student Edition* and *MathMaster 2.21* were demonstrated, they were unavailable

to the students working through the modules due to copyright restrictions upon their use (these restrictions were later relaxed for *MathMaster* by the author, allowing it to be used freely in subsequent data collection activities).

After the class sessions introducing the software and materials, students were allocated six weeks in which to complete the assessment tasks associated with the project. They were free to access the materials on the available computers at times of their own choosing.

Group B: The second group of preservice teachers participated in a twelve hour unit specifically on the role of technology in algebra, taught by the researcher over four three-hour sessions. The course outline and assessment requirements are included in Appendix D, and deliberately favoured an exploratory approach, emphasising use of the tools within problem-based situations in addition to instructional content-based activities. Computer algebra facilities were available for this group (in the form of *MathMaster 2.21*) in addition to the modules of *Exploring Algebra* and the multiple representation tool, *xFunctions 2.2*.

References were provided to accompany each week's theme, and students were instructed to complete the assessment requirements at times convenient to them, over the duration of the unit and the four weeks following. Each of the five explorations specified in Assessment Task 1 was allocated a weighting of 12%; the other two modules were weighted at 20% each. The weighting deliberately stresses the free exploration component of the task, and encouraged students to engage actively with both the tools and the instructional materials.

As with the secondary students, ethical considerations constrained the gathering of research data in both preservice teacher groups. Once again, the priority for both groups was the learning experience, and tasks which served only a research function were largely inappropriate. A number of the students in Group A did, however, choose to work through some of the Research Questions available within the materials, providing data revealing of several aspects of their thinking about mathematical and pedagogical aspects of algebra. This group was, however, restricted in access to computer algebra software, since multiple versions of the commercial tools were not available and, at this stage, permission had not been granted to use the shareware program, *MathMaster 2.21* (such permission was subsequently granted by its author). Group B had no such software limitations and had ready access to both *MathMaster* and *CoCoA* which both offer basic algebra capabilities.

Each group, then, offers particular strengths in terms of the research data provided. Group A offers rich description of the pedagogical aspects of algebra learning, but is less strong in the mathematical aspects. Group B, on the other hand, engaged more vigorously in tool-based mathematical exploration, but offered less in the way of pedagogical data. Together, the groups complement each other well and provide detailed data regarding the ways in which these preservice teachers chose to engage in the use of mathematical software within an algebra learning context.

Analysis of Data

The varied and extensive data derived from this computer-based design were, in almost all cases, in text-file format, ready to be read directly into *NUD•IST* (Richards and Richards, 1993), the qualitative analysis software tool chosen for analysis. The use of computers as tools for qualitative data analysis is now widespread, since they provide unique and powerful means of working with relatively large amounts of textual data (and, increasingly, other formats including graphical, audio and video). *NUD•IST* is unique in providing a wide range of search and retrieval functions which may be applied, not only to the data itself (referred to in the program as the *Document System*), but also to the categories created by the researcher which are organised to form what is termed the *Index System*. The program encourages and rewards the creation of a two-dimensional tree, with each new category becoming a *node* on the tree which must be located in relation to other nodes already in existence.

This formal structure of the *Index System* appears highly compatible with the Grounded Theory approach, which utilises as its main tool for data analysis what is termed the *constant comparative method* (Strauss and Corbin, 1991, p. 62). As each new category is created it must be compared and contrasted with existing categories - a feature deliberately encouraged by *NUD•IST*. The *Index System* itself is in a constant state of flux throughout the data analysis process (which begins with the first collection of data and continues throughout the period of the project). It is intended that the *Index System* reflect the state of the researcher's organisation and conceptualisation of both the data and the abstract categories which arise from it.

In the present study, categorisation of the data began early in the data collection process with the identification of concepts arising directly from the data. This can rapidly lead to a proliferation of categories - over ninety categories emerged quite rapidly from the first coding process using a software package which did not support the tree-structure of *NUD•IST*. A significant advantage of this structured approach is that it actually helps to minimise the growth of new codes, since each is compared and contrasted and *situated* in relation to others.

An early form of Index System developed for this study included codes for the several theoretical positions outlined in the Introduction - Constructivism, the SOLO Taxonomy, van Hiele and Vygotskian categories. This approach was later rejected as incompatible with a Grounded Theory approach, since the theoretical structure must arise from the data, rather than being imposed upon it. This is significantly different from more traditional research approaches which are based upon theory *verification* rather than theory *generation*, as is proposed here. Alternative theoretical positions are more important at the end of the analytic process than the beginning. They serve then to support and perhaps to generalise the theoretical position which has been developed.

As these concepts are organised and refined, common themes are recognised and serve as the basis for new concepts, emerging at higher levels of abstraction and becoming increasingly *theoretical* (a process aided significantly by the keeping of journal notes and *memos* as reflective devices which assist in the movement to levels of increasing

abstraction from the data). The resultant grounded theory is derived from the data by a cyclic process of *organisation* -> *abstraction* -> *theorising* -> *verification* -> *refinement*. These last stages of verification and refinement return the researcher to the data once again to reassess the categories which are contributing to the emergent theory. The data collection process itself in the later stages will also be driven by the theoretical perspective which is being developed and, increasingly, the verification stage will involve both a return to existing data and the gathering of more focused sources of information.

Strengths and Limitations of the Research Design

This study is cognitive and naturalistic in nature, seeking to elicit aspects of mathematical and pedagogical *thinking* by individuals learning algebra in a tool-based context. Since thinking itself is not open to scrutiny, it must be made explicit through consideration of the twin elements of *action* and *language*. The research instrument was designed to capture as much as possible of these elements within a particular algebra learning context. The attempt to maintain such a context in as naturalistic a mode as possible imposed certain quite significant limitations upon the design, while at the same time offering the potential for the collection of data which accurately and extensively reflects the concerns and realities for those engaged in the processes of algebra learning.

Particular limitations of the research design may be recognised as deriving from the following factors:

- *participant responses*
- *sample limitations*

- *generalisability of results*
- *researcher bias, and*
- *artificiality of the context.*

All but the last of these are common to most qualitative research studies and are frequently cited as criticisms of the Interpretivist paradigm. Any study which purports to study *thinking* must confront the most obvious hurdle - thought processes themselves are inaccessible and must be studied by inference. In particular, the assumption that language provides an adequate representation of thought may not be brushed over lightly. It was precisely this problem which Vygotsky confronted in his classic text, *Thought and Word* (1962). While denying that one is in any way an accurate mirror of the other, his fundamental thesis revolved around the links between thinking and word *meaning* (Vygotsky, 1962).

The meaning of a word represents such a close amalgam of thought and language that it is hard to tell whether it is a phenomenon of speech or a phenomenon of thought. (p. 120)

Such has been the approach adopted in the current study. The responses elicited from the participants are seen as providing critical insights into their thinking. The more varied the response (verbal, visual, tactile) the more rigorous the connection. In the final analysis, however, we make one unavoidable assumption - that our informants are speaking the truth, providing an accurate description of their own understandings and perceptions. The relationship of the students with the researcher, built up over an extended period, coupled with the face-to-face contact of the tutorial situation, serve to alleviate the concerns for this group. The preservice teachers, however, have little or no relationship with the researcher, nor was anyone watching over their

shoulders as they interacted with the software and responded to the prompts and probes - how then can their responses be trusted?

Two considerations are relevant here. The first was the nature of the research process as a significant part of their course assessment (occurring in the latter part of their studies when successful completion must hold a high priority). The second is the nature of the responses - these varied from minimal and obviously hasty answers from some, to detailed and extended tracts from others. Since the research process was a relatively time-consuming one for these students, it is assumed that those who took the time to provide lengthy and considered responses were being accurate in these. For this reason, six preservice teachers from Group A (four female, two male) who provided the most detailed responses and completed a significant part of the supplementary research questions were chosen as the principal informants. While the remaining twelve students provided useful comparative data, the emphasis in the analysis lies with the chosen six. Similarly, of the eight students in Group B, two who provided the least detail were excluded from the primary informant group.

This leads to the second recognised limitation of the study, the limitations of the sample. The absence of a third obvious group of participants - practising teachers - was a deliberate choice made late in the research process. Although their insights and perceptions would have provided valuable data, it was decided that the focus should be limited to algebra *learning* situations. If the results are to inform our understanding of the role of computers in mathematics learning and so to improve teaching practice, then we must begin by understanding how students *learn* with technology; only then can we begin to explore

how teachers may best *teach* with it. The inclusion of preservice teachers was considered appropriate, since they, like high school students, are engaged in the learning of algebra, although with a different emphasis.

While the various participants chosen represent cross-sectional groupings which add a significant degree of variability to the data, they can in no way be considered representative of students or preservice teachers in general. As a case study, however, their responses provide insights and perceptions valid for themselves which in turn may serve to illuminate our understanding of others in algebra learning situations. Generalisability of results, then, lies in the eye of the beholder. The detail, variability and accuracy of the data serve to inform others who may then seek to discover the extent to which the findings apply to their own situation. A good grounded theory remains just that - a theory which describes and informs by the systematic and insightful way in which it was derived from a particular data set, and then invites others to provide their own experiences which may support or provide counter-examples. The latter, of course, serve a vital role in developing any theoretical position.

Grounded theory demands a specific recognition of the stance of the researcher, known as “bracketing”. Within qualitative research design, the researcher is part of the phenomenon being studied and must be aware of the values and perceptions which this inside position brings to the enterprise.

If such bracketing is not done, the scientific enterprise collapses, and what the sociologist then believes to perceive is nothing but a mirror image of his own hopes and fears, wishes, resentments or other psychic needs; what he will then not perceive is anything that can reasonably be called social reality. (Berger and Kellner, 1981, in Hutchinson, 1988, p. 130)

Bracketing in this study is achieved through journal-keeping and research memos, kept throughout the course of the project and, specifically, through the researcher himself completing each of the Research Questions described above (providing a detailed profile), along with several open-ended mathematical tasks using available tools.

The research instrument itself provides perhaps the greatest limitation and at the same time the greatest strength of the research design. Removed as it is from the most common algebra learning mode - the mathematics classroom - it offers instead a focus upon the individual interacting with both mathematics and technology. While in no way denying the importance of social interaction in the learning processes, such a restriction potentially offers a clearer view of the issues in question. While the instructional modules created for this task may be imperfect in their realisation of the ideals of algebra learning emerging from research, they serve adequately as a starting point, or springboard, by which users may be directed and encouraged in their use of software. It is the software use which is the focus, not the nature of the algebraic environment. Further, the inclusion of open-ended problems is likely to be far more revealing of strategies of software use than the limited instructional sequences provided.

Specific strengths of the research design may be recognised in relation to the following factors:

- *Immediacy*
- *Accuracy*
- *Minimal obtrusiveness*
- *Context*
- *Reliability and Portability*

Within the context of the limitations outlined above, the computer-based learning and research environment developed for this study appears to offer certain clear advantages over alternative designs. Since the user is actually involved in an algebraic learning situation when prompted for responses and descriptions, the *immediacy* of the design potentially offers increased accuracy which may be less certain within, for example, *stimulated recall models*. These assume that certain prompts (especially audio and video recordings) will inspire accurate and detailed recollection of the thinking which occurred previously. The model developed for this study accurately captures the *flow of interaction* which is the principal object of study.

At the same time, the research instrument is designed not only to capture the responses to verbal prompts, but to monitor unobtrusively the physical interactions of individual with computer - which options are chosen, which buttons are pushed, the time taken at each card. All are accurately recorded to provide a detailed session record. In this way, the design is as *unobtrusive* as is possible within a legitimate learning situation, and certainly far more so than video and audio recording devices which intrude significantly upon the confidentiality of the learning experience.

As stated previously, the software tools under consideration in this study are always used within a particular learning context. In addition to providing instructional models developed from the results of research, the design offers an open-ended environment to which the learner may bring problems and queries of their own. This proved to be a most valuable option for the secondary students in particular, who frequently used the available utilities in a problem-based rather than

instructional situation. Senior students especially preferred this mode in preparation for assessment tasks and examinations.

Possibly the most valuable feature of the research instrument lies in its *portability*. To a large extent, it operates independently of the researcher and offers the attractive option of being used at any time convenient to the participant. More importantly, it offers a level of objectivity in terms of the data collected which is more often associated with surveys and questionnaires, while retaining the flexibility and open-endedness of an informant interview.

The research design, then, offers a model of data collection and analysis which is sufficiently dense, systematic, valid and reliable to serve as a basis for the development of the grounded theory proposed.