

INTEGRATING TECHNOLOGY IN THE MIDDLE SCHOOL

Stephen Arnold

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Dr Stephen Arnold has been a teacher for 25 years, and has been actively using classroom technology for over 15 years. In addition to teaching mathematics, science and technology in a range of New South Wales high schools, Stephen has been involved in teacher education at James Cook University and the University of Newcastle. For the past five years, Stephen has been applying what he has learned by developing new ways of integrating technology into the classroom.

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Graphic Calculators

Years 5-8

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INTRODUCTION

The teaching and learning of mathematics has been forever changed by the influence of technology. No longer a static body of knowledge and skills to be memorised and practised, mathematics is now generally recognised primarily as an ongoing deliberate activity: a search for patterns and relationships. Whenever we are exploring regularities, keeping an eye out for recurring themes, observing predictable behaviours – we are doing mathematics. Access to appropriate technology offers new tools for this search, just as it supports students as active and purposeful learners.

A central theme of this book lies in the recognition of individual differences among our students: well-designed learning tools can play a crucial role in providing students with a means of working at their own pace, both in order to work towards the skills and understandings already achieved by the majority of their peers, and further, in some cases, to work beyond these achievements. Such tools have the potential to support students in their learning, scaffolding them in such a way that they gradually become independent and confident in the demonstration of required skills and understandings. They provide stepping stones by which able students may see beyond their current practice and reach further than they could when unaided. Some technological tools make possible for both teacher and learner things that could not be achieved in any other way.

This book seeks to support teachers in their uptake of technology in support of the new modes of teaching and learning which are now widely recognised for mathematics. The major focus of this book will be upon the software Apps developed by Texas Instruments for the TI-83 Plus calculator. These powerful tools offer learning possibilities previously restricted to computers, including such important mathematical tools as spreadsheets and interactive geometry software, well-designed learning tools aimed at specific syllabus areas, interactive teaching sequences designed by teachers – all in addition to the powerful mathematical capabilities of the graphic calculator. While teachers and students can and should be encouraged to make use of computers in their learning of mathematics, there is currently no more appropriate nor powerful tool for mathematics learning available to schools.

Consider the possibilities!

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1. NUMBER

Overview: Whole Numbers and Integers

At the end of primary school, students are expected to be able to order, read and write numbers of any size using place value. They can interpret and express numbers using expanded notation and locate position on the number line (including negative numbers). They select and apply appropriate mental written or calculator strategies for addition, subtraction, multiplication and division of whole numbers and use formal written algorithms (limited in the case of multiplication to two digits and for division to a single-digit divisor).

With the commencement of high school, they begin to extend their knowledge of number properties and relationships.

Technological tools are well suited to explorations of number; they offer students opportunities for calculation support and pattern recognition that extend and enhance their natural capabilities. With the addition of well-designed software tools with appropriate pedagogical themes, students may be further empowered to investigate and learn both concepts and skills, which form the basis for much of their further study in mathematics and beyond.

APPLICABLE APP: *Number Curiosities*

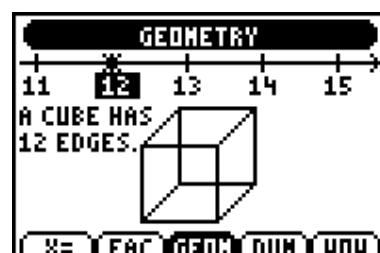
This concept app displays the numbers from 0 to 25 at the top of the screen on a number line. Students may use the arrow keys to move backwards and forward along this number line, and so may use the app to support addition and subtraction activities.

Linked to the five blue “soft keys” below the graphing screen are menu options which allow the user to explore different properties of each number.

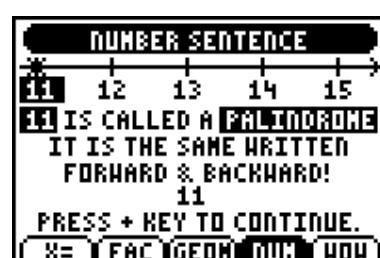
1. **X = :** The first menu option allows the user to move quickly to any number by entering it (if it is a single digit number, press ENTER; if a two-digit number the app will move directly to that value after entering the second digit).



2. **FAC:** This option displays the factors of the number if it is composite (as shown above). If the number is prime, it simply displays the word “Prime”.
3. **GEOM:** Geometric properties of each number are a source of interest explored by the third menu option.



4. **NUM:** This option displays number relationships associated with the chosen number. There is also an invitation to investigate such properties further. In the case of the number 11, for example, the properties of palindromic numbers are explained and explored over several pages of information and questions.
5. **WOW:** This one is just for fun, linking the number to some real-world application (e.g. *There are 11 players on the field during soccer and football games*).



LEARNING ACTIVITY N1: ORDER AND DISORDER

AIM *To build student understanding of the number line as a tool for representing, ordering and for operating upon numbers.*

METHOD *Physical involvement remains one of the most powerful tools for learning, especially in cognitive areas such as mathematics. This activity engages students in ordering and operations activities, which will be remembered for a long time because they were involved in a physical way.*

PROCESS: Stage 1: Ordering (Up to 10 minutes)

1. Allocate numbers from 0 on to each student in the class. This may be done using prepared cards, or by some other method.
2. Have your students form a continuous line in numerical order (this may need to be done outside the classroom!)
If you are brave, you might ask that this be done without speaking and without letting anyone else know your number!
3. Once they are settled, have them all sit down. Ask then that each in turn calls out their number (and make any necessary adjustments!).
4. Ask students to stand if they are *less than 10*. Sit down.
Greater than 5. Sit down.
Stand up if you just stood up for both of those questions. Explain in your own words why.

Continue this exercise until you are satisfied that all students are comfortable with ordering whole numbers.

Stage 2: Addition and Subtraction (10 minutes)

1. For this activity, use a ball or some other visible object that students will pass from one to the other. You may even like to take the last few students and make them “movers”.
2. The rules are simple: the ball or mover always begins at 0. As the mover passes a number, that person calls that number out. The person where the mover stops must stand up.
Begin with a question such as **2 + 3**.
If necessary, explain that the mover begins at 0 and moves two places to the right (“0”, “1” and “2” call out their numbers). **Plus 3** signals that the mover continues another 3 places to the right (“3”, “4” and “5” are called out). The person representing the number “5” stands up.
3. Continue to ask simple addition and subtraction questions until you are satisfied that students are comfortable with the process.
4. If you are brave and your students are ready, try re-allocating the **zero** position to someone else in the middle. Continue the game as students work out their new positions relative to the zero person. Do this several times.

Stage 3: Number Properties (10 minutes)

1. Reallocate the zero person back to the start, and have students use their graphic calculators to run the *Number Curiosities* App. For classes with more than 25 students, ask the numbers greater than 25 to choose their favourite number and to go and sit with the person in that position.
2. Have each student press the FAC key and move to his or her number. Have all the PRIME numbers stand up. Ask for observations from the class. Ask the prime numbers how they feel? (Special, perhaps?) Ask if there are any COMPOSITES with more than 3 factors. Who has the *most* factors? What about *repeated* factors: the same number over and over? Whose factors can be divided equally in two? What is special about these numbers? Continue if you wish (You may want to talk about *abundant* numbers, with lots of factors – such as 12, where $1 + 2 + 3 + 4 + 6 > 12$, and *deficient* numbers, with few factors – such as 9, where $1 + 3 < 9$. Even consider *perfect* numbers, such as 6, where $1 + 2 + 3 = 6!$)
3. Now have each student press the GEOM key and ask all the odd numbers to stand up. How do they feel, being odd? Ask all the *perfect squares* to stand up. What was special about their factors? What about the *triangular numbers*? What is the pattern here?
4. Have students compare notes with those around them. Use the app to find out other interesting things about their number.

EXTENSION and ASSESSMENT

On returning to class, students should record what they have observed and learned from this exercise. This form of simple journal-keeping should be a regular part of their mathematics lessons, reflecting on what they learned, what they most enjoyed, how they might have learned more.

Assessment: Prepare a report on *your favourite* number, recording as much as you can about that number and what makes it interesting.

Alternative Task 1: Prepare a class debate: *That even numbers are more interesting than odd numbers.*

Alternative Task 2: Complete each of the *Number Curiosities* headings for the numbers from 26 to 30. Make this a research assignment which may be completed individually or in groups.

Overview: Formal Operations

The introduction of formal written algorithms (particularly those for multiplication and division) are commonly delayed in recent syllabus documents. Strong research evidence combined with the wisdom of practice indicates that students need to be well-established in their informal intuitive (common-sense) methods before being able to understand the processes and requirements of the more formal approaches. Too often in the past, the acquisition of written algorithms actually overwhelms previous understandings and students are left blindly following rote procedures. All too often, errors result from these algorithmic methods when previous informal methods would have been more successful.

The introduction of written algorithms in the later years of Primary school implies, too, that many of the students beginning high school are likely to have poorly developed skills in this regard. There will be increasing pressure on high school teachers to explicitly deal with these operations, and to cater for the wide range of skill levels which students will bring with them. The use of appropriate technological tools in this regard will be of significant benefit, since students will be able to work at their own pace, starting from their own level of achievement and practice for as long as each requires.

LEARNING ACTIVITY N2: A FORMAL INTRODUCTION

APPLICABLE APP: *Math By Hand*

$\begin{array}{r} 256 \\ + 174 \\ \hline 10=6+4 \end{array}$
[BACK] [NEW] [NEXT STEP]

$\begin{array}{r} 256 \\ + 174 \\ \hline 10=6+4 \\ 120=50+70 \\ 300=200+100 \\ \hline 430 \end{array}$
[BACK] [NEW] [REPEAT]

This concept App is in many ways a teacher's dream, as it patiently and carefully takes students step-by-step through the formal procedures for addition, subtraction, multiplication and division. Three levels are available to cater for students of differing confidence and skills: the step-by-step process illustrated above, a fast solve option and then a problem solving level, in which the questions generated involve various missing pieces, and students must "work backwards" in order to find these values.

$\begin{array}{r} 4 16 \\ - 2 5 6 \\ \hline 1 8 9 \end{array}$
UNITS PLACE REGROUP IF NEEDED, THEN SUBTRACT!
[BACK] [NEW] [NEXT STEP]

$\begin{array}{r} 1 14 16 \\ - 8 5 6 \\ \hline 6 7 \end{array}$
HUNDREDS PLACE SUBTRACT!
[BACK] [NEW] [NEXT STEP]

The subtraction algorithm provides instructional comments for each step, accompanying and explaining the processes involved. Note, too, that attention is drawn to the

place value: units, tens and hundreds.

The multiplication algorithm similarly draws the student's attention to those particular parts of the question involved at each step, and makes explicit their values in the ongoing operation. Students enter their own question values, and so can begin as simply as they require and proceed at their own pace. They are free to use this scaffolding with questions at any time, and might be encouraged to use this tool rather than the single solution offered by the calculator when confronted by such a question. Division is treated carefully, but probably will require some early teacher input in order to explain the beginning steps. The option to present the result using simplified and unsimplified fractions, as shown, is another useful feature.

$\begin{array}{r} 856 \\ \times 24 \\ \hline 24 = 4 \times 6 \\ 200 = 4 \times 50 \\ 800 = 4 \times 200 \end{array}$
[BACK] [NEW] [NEXT STEP]
$\begin{array}{r} 19 \\ 8/156 \\ \hline 80 \\ 76 \\ 72 \\ 4 \end{array}$ FRACTION: $19\frac{4}{8} = 19\frac{1}{2}$
[BACK] [NEW] [REPEAT] [FRAC]

This learning tool speaks for itself in terms of its applications to the classroom. It is a well-designed instructional aid which lends itself to individual use by students over a period of time. In the early stages, it can also be used with a ViewScreen™, allowing the entire class or particular groups to be taken through the steps involved.

Overview: Fractions, Decimals and Percentages

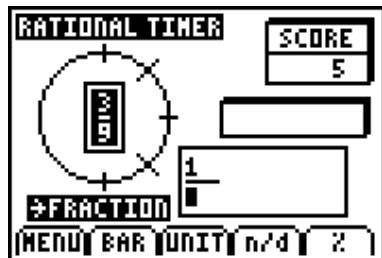
At the end of Primary school, students have learned about the various representations of rational numbers and have some facility in moving between these representations for commonly used fractions. They are able to add and subtract with decimals and a limited range of fractions and percentages (those with related denominators).

At the commencement of high school, they are expected to further strengthen their representational facility and to be able to move with confidence between these forms, using this as the means of ordering and interpreting the actual proportions of the numbers involved. They also complete their operational training, learning to manipulate and perform all four operations with all forms of rational numbers.

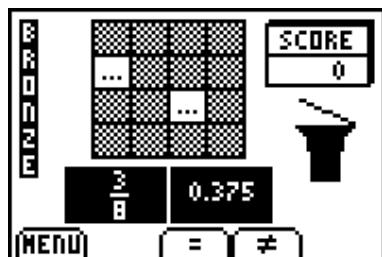
LEARNING ACTIVITY N3: MIXING YOUR RATIONALS

APPLICABLE APP: *Rational Rampage*

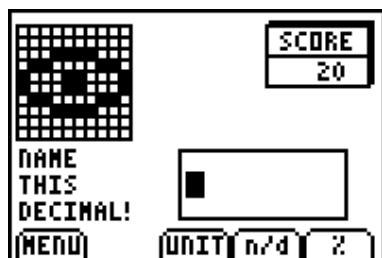
Children of all ages learn best when they are motivated, and some form of game or competition is always a good place to start. *Rational Rampage* offers three different game formats by which students learn to recognize and move between the three representations of rational numbers: fractions, decimals and percentages. Different skill levels are available (Bronze, Silver and Gold) and scores and high scores are kept.



In *Rational Timer*, students are challenged to convert between the forms while a timer ticks away in the background. Using the menu keys at the bottom of the screen, they are able to enter fractions (n/d), mixed numerals (unit), percentages (%) and even repeated decimals (BAR).



Rational Match is a version of the traditional game of “Concentration” where rationals in different forms are hidden behind squares. Students may show what is behind two squares at a time, and then decide if they are equal or not. They must then remember where the different values are hidden in order to win. This is a great game played in pairs or with the whole class.



Finally, *Percent Square* uses a 10 x 10 grid with a random array of shadings to offer students the opportunity to test their understanding of fractions, decimals and percentages as statements about proportion: What fraction of the grid is shaded? Once again, the questions require results in each of the three forms and students are frequently challenged, not only to correctly perform the conversion required, but also to look for patterns which make the counting process easier!

This App is perfectly suited for a variety of classroom applications: individual practice, small groups or even whole class using a ViewScreen™ and overhead projector. Scores are kept and students (and their teacher, if desired) can retain a record of progress.

Each of the Apps described has, built into its essential structure, the important element we describe as *working mathematically*. Each goes beyond a simple task to engage the student at a variety of levels and across a range of mathematical activities. Using such technology ensures that at all times, students are active participants in their mathematical learning – never spectators.

LEARNING ACTIVITY N4: DEFENDING DECIMALS

APPLICABLE APP: *Decimal Defender*



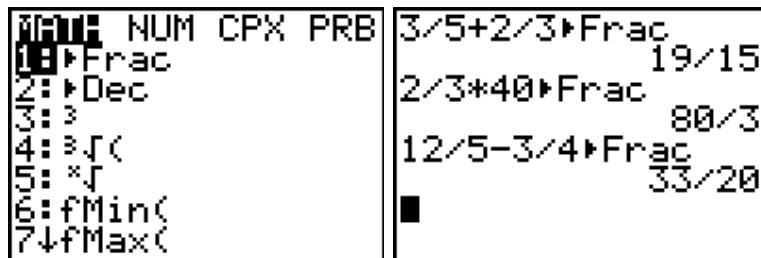
This App offers a simple but powerfully effective format for learning about placement of the decimal point for multiplication and division by powers of 10. It provides an Microsoft Excel®lent introduction to decimals and place value and provides students with a fun way to master the skills required.

The game format involves shooting the decimal point into the correct position in order to repel an alien attack. Left and right arrows allow the user to choose the correct position, and the up-arrow fires. If the answer is incorrect, the aliens break through and invade the planet! Clearly, this will be popular with boys, but my experience with girls indicated that they were just as competitive and found it equally enjoyable.

Scores are kept, high scores are recorded and students and their teacher can keep track of progress. My son in Year 5 had not encountered this aspect of decimals prior to playing *Decimal Defender*, but he was soon expert at multiplication and division using powers of ten!

LEARNING ACTIVITY N5: FRACTION OPERATIONS

CALCULATOR APPLICATION



The **MATH** menu on the TI-83 Plus calculator contains a wealth of useful tools, but the very first one in the list is well-suited to further work by students on operations with fractions.

The **FRAC** key expresses the result as a fraction in simplest form. This is an Microsoft Excel®lent tool for students learning to add, subtract, multiply and divide with fractions.

A third-party App called *PrettyPrint* (freely available from www.softtheiss.com) adds the further capability of displaying mathematical symbols in correct two-dimensional format. For students who may be distracted by the single-line display of fractions available from the home screen, this offers a more appropriate display.

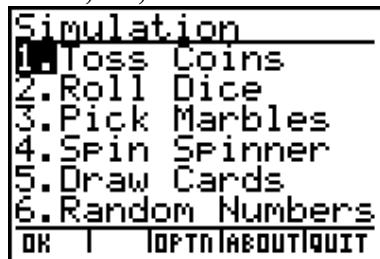


Overview: Chance

It would be hard to conceive of students at almost any age working mathematically without some reference to notions of likelihood and chance. References to “fairness” feature as a regular part of any playground. While the more formal treatment of probability is not commenced until later in high school, informal considerations can and should abound in earlier years. At these stages, all probability should be experimental, as students are led to appreciate the sometimes elusive links between simple events and the expectations that may accompany them. Links between concepts of chance and the gathering of data should also be made explicit early on. It would be difficult to consider concepts of chance without some use and reference to at least simple statistical methods of gathering, organizing and displaying data.

LEARNING ACTIVITY N6: WHAT IS THE CHANCE OF THAT?**APPLICABLE APP: *ProbSim***

NS2.5, 3.5, 4.4



There is absolutely no argument that students need to engage physically with as wide a variety as possible of chance situations before turning to computer- or, in this case, calculator-based simulations. Tossing coins, rolling dice, picking marbles, numbers and other objects, drawing cards and even spinning spinners: there is much to be learned from the physical engagement in such activities. Tempting though it might be for ease and convenience or even for a quieter

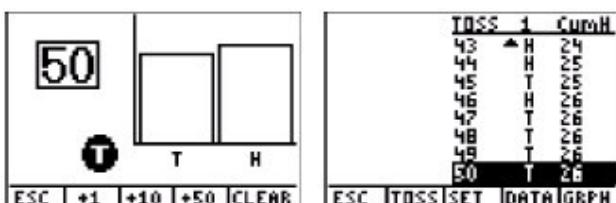
classroom, it is vital that students of every age engage in such activities to lay solid foundations for their future work in probability. Further, while such “classroom experiments” offer clear and easily recognizable links to simple number statements, it is also critically important that students engage with “real world” aspects of this important aspect of mathematics: *How often do you get caught at those traffic lights? Why do they always seem to be out of my size? How can I increase my chance of winning in the school Easter raffle?*

Once such foundations are established, then the world of simulation comes into its own. Students may quickly and easily generate vast amounts of experimental data which can be readily displayed in a variety of ways. Using the *ProbSim* app, they can control and manipulate the experiments, too, bringing a richer and deeper understanding of the relationship between the number of ways in which an event can occur and the frequency of a “win”.



user may control various aspects of the experiment. Here we decide if we want to toss 1, 2 or 3 coins, and other variables. Pressing **ADV** “advances” on to the next screen, where the

weights of the coins may be varied. In this initial experiment, we will leave the coins equally weighted and so with an equal probability of head or tail.



the result of 50 trials. If desired, students may select another 50, and so on to observe the long term results. Finally, pressing **DATA** allows students to inspect the actual results, along with a cumulative count of the number of heads obtained. These results may be stored in lists for further study.

This simple experiment demonstrates both the ease of use and the clarity of the display for this app. It is suitable for use with students from quite early years through to seniors and offers a wealth of investigations and experiments which will build student understanding of probability concepts.

LEARNING ACTIVITY N7: ALL AT SEA CALCULATOR APPLICATION

(with thanks to Doug Clarke)



What percentage of the world's surface is covered by land?

There are many ways to seek an answer to this question (including perhaps using the *World Geography* App and adding the areas of every country) but most are tedious and time consuming, or involve simply finding a reference book. A nicer method uses sampling.

Use an inflatable globe of the world. Have students take special note of, say, the thumb of their left hand. Throw the globe to any student in the class and ask them to take note of whether the their left thumb landed on land or sea. If land, then enter 1 into **L1**; if sea, then enter 0.

Have the student with the globe throw it to someone else. Again, record the result with a 1 or a 0. Stop after 10 throws and observe how many “1”s are in the list – or go to the Home Screen and enter from the **List** menu **sum(L1)/dim(L1)**. (**sum(** may be found as item 5 under **MATH** on the **List** menu; **dim(** is item 3 under **OPS** and gives the *dimension* or number of items in the list). Record the percentage shown.

Continue the game for another ten throws, again recording the percentage total. As the total number of samples grows, students should observe this percentage approaching a value which gives an estimate of the percentage of the earth’s surface covered by land.

Discussion should follow concerning this effect: the greater the sample size, the better the estimate. Discussion should also focus upon limitations of the method: what factors may reduce the accuracy of the result?

Ask the students to consider other sampling methods, which might be used to answer this question.

2. PATTERNS AND ALGEBRA

Overview: Introducing Algebra

The study of algebra begins with number. Students must understand numbers and their relationships before they can understand the generalized form, which is algebra. Tables of values provide an ideal tool for such explorations and concept building. The move, then, to symbolic forms must be managed carefully if students are to avoid a rote-learning approach, which denies them access to the power and applications of algebra later. Just as with number, technology can provide a supporting mechanism by which early learning can be scaffolded and built upon solid and versatile foundations. The dynamic and often interactive technological tools now available are ideally suited to this domain of learning.

LEARNING ACTIVITY A1: FROM NUMBER TO SYMBOL CALCULATOR APPLICATION

3	3
Ans+2	5
	7
	9
	11

The large home screen of the graphic calculator is ideally suited to the study of number patterns. As shown, begin with a number (say, 3). Perform an operation (*add 2*) then continue to press **ENTER** and this last operation will continue to be carried out: the device takes the last answer (ANS) and adds 2 each time. The pattern emerges naturally. For stage 3 students, this exploration of the effects of a single operation is sufficient for their introduction to

patterns and algebra. In fact, it is the *putting into words* of this action that is critical in these early stages. Algebra should be *meaningful* to the learners. In this case, the pattern describes a process of *adding 2*.

The same pattern can be readily modeled in another way, too, using the available technology: the **Table of Values**. Long before they are introduced to the symbolic forms which drive the table of values, students may be introduced to it as a notation for recording and observing patterns. However, beware of the enormous magnitude of the conceptual shift between the *vertical* relationship of the first description (in which each of the numbers in a single line is derived from its predecessor) and the *horizontal* relationship of the second (where the numbers in the pattern are linked to the counting numbers in such a way that a rule which defines this relationship may be discovered). In a very real sense, these two representations capture the difference between the study of patterns in the Primary school and that which marks the beginning of the formal study of algebra in the secondary school.

4	4
Ans*2-5	3
	1
	-3
■	-11

X	Y ₁
0	3
1	5
2	7
3	9
4	11
5	13
6	15

X=0

Teachers should make frequent use of both representations: begin with students exploring home screen operations with repeated actions: *What happens if I multiply by 3?*

Subtract 4? Divide by 2? Note that such explorations quickly lead students into both rational and negative numbers, in a way which retains meaning for them. Stage 4 students should then be encouraged (even expected) to discover the effects of *multiple operations*.

X	Y ₁
0	0
1	1
2	2
3	3
4	4
5	5
6	6

X=0

X	Y ₁
0	0
1	2
2	4
3	6
4	8
5	10
6	12

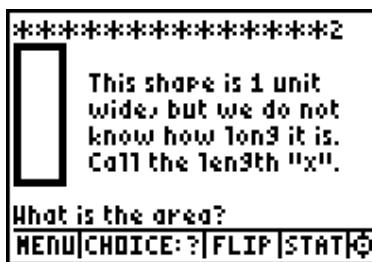
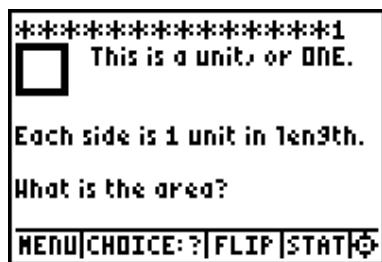
X=0

Begin with 4.
Multiply by 3 and subtract 5.
Repeat this operation.
Record the pattern using the Table of Values format
What do you observe about this pattern?

The table of values display invites discussion concerning the **X** and **Y** values. Using simple examples ($y = x$, $y = 2x$) students can be led to see the relationship described and how it is expressed in the appropriate number pattern. As shown below, students may be encouraged to explore the **Y=** menu and the effects that the rule entered has upon the pattern displayed. At every stage, students should be asked to *write down their observations*: to put into words what they think is happening. This enforced use of language helps to retain meaning through the process of symbolizing. We are leading students to the stage where, given a (linear) pattern of numbers, they can draw from this the symbolic rule ($y = ?x + ?$).

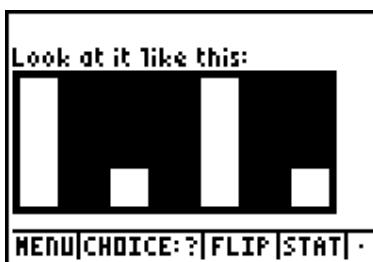
LEARNING ACTIVITY A2: FROM SYMBOL TO MEANING

APPLICABLE APP: STUDYCARDS



It is one thing for students to link the familiar operations to number patterns (*2 times something plus 3*). The *action* of the operations remains firmly linked to numbers, so the student remains on solid ground, conceptually.

It is an entirely different order of cognitive magnitude to talk about (and eventually manipulate) an *algebraic object* such as $2x + 3$. The first is a *process* conception; the second, an *object* conception. The problem with traditional approaches to algebra lay in students moving too quickly to this second form, and losing the tenuous grasp they had upon the first. Further, since the second form was largely meaningless to them, this led to students adopting a rote learning approach to algebra, trying to become proficient with rules and procedures which, in the end, remained quite mysterious. *Fruit salad* approaches to algebra (in which *a* stood for *apple*, *b* for *banana*) seemed quite appealing in that they seemed to offer short-term solutions which students understood concerning ways in which these variables could be combined and manipulated. Sadly, these methods failed in their internal logic and, in fact, resulted in serious inconsistencies which later were difficult to overcome.



A more internally consistent method uses areas of shapes to represent variables and numbers. As shown above, a unit square becomes a “1”; a rectangle 1 unit wide but of unknown length becomes an “ x ” (since its area is x square units). These shapes can then be used to lead students to logical and consistent procedures for working with algebraic variables.



This is a surprisingly robust model. Not only do students readily recognize patterns as representing $2x + 2$, but they can be led to see ideas associated with grouping: in this case, $2(x + 1)$, or “two lots of $x + 1$ ”. Further, introducing shaded shapes to represent negative quantities allows the model to

extend further (the example shown can be seen as either $-3x + 4$ or, more simply, $4 - 3x$. The use of a large square ($x \times x$) introduces an x^2 symbol, and even quadratic factors and perfect squares may be introduced.

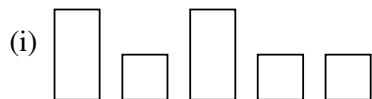
Students may cut out their own shapes and use them until they have a strong mental concept and are able to carry out quite involved simplifications. Questions such $2x - 3 - 4x + 5$ prove easy following this approach. Even a problem such as $2(x - 1) + x^2 - 3x$ are accessible. Because they visualize shapes for numbers and x 's and even x^2 's, students using this approach do not confuse the terms and rarely make errors of simplification.

The *StudyCards* app was ideally suited to presenting this approach to students, allowing as it did a mixture of text and simple graphics, as well as a multiple-choice selection facility. It has proved suitable as both an individual and a whole class activity. The stack *AlgIntro* may be downloaded as part of a supporting file collection from

<http://homepages.ihug.com.au/~arnolds/alg83.zip>.

WORKSHEET STARTER:

1. Write the names (“ x ”, “ $-x$ ”, “1”, “-1”) under the symbols and use these to simplify the algebraic expressions:



2. Draw the shapes for each algebraic expression, then simplify.

(i) $x + x + 1 + 1 + 1$

(ii) $2x - 1 + x + 3$

(iii) $2(2x - 1)$

LEARNING ACTIVITY A3: MANIPULATING SYMBOLS

APPLICABLE APP: SYMBOLIC AND PRETTY PRINT

The nice people at *Detached Solutions*

(<http://www.detachedsolutions.com>) have written some wonderful Apps for the 83Plus, including one called *Symbolic*. As the name implies, this adds basic symbolic algebra capabilities to the 83Plus: in particular, a **SIMP** command to perform simplification and expansion of algebraic expressions, and a symbolic differentiation command, **D**. These functions operate on string variables: i.e. they require inverted commas around the input. This makes them a little awkward to use, and the output tends to look a little messy.

$$2 \cdot (2 \cdot x - 1) = 3 \cdot x + 5$$

◀ ▶ ⚡ OK

Fortunately, when combined with another App, called *PrettyPrint* (also available from *DetachedSolutions* but written by *SoftTheiss* at <http://www.softtheiss.com>) this problem disappears. *PrettyPrint* does what it promises and presents algebraic expressions in correct formatting, making it easy for students at all levels to interpret their algebraic output. These two apps can work seamlessly together, but to make the process even easier, I wrote a couple of TI-BASIC programs which offer a simple front-end for teachers and students who would like to add some CAS capabilities to their 83Plus calculators.

$2 \cdot x \cdot (1 - 3 \cdot x^2)$

$2 \cdot x - 6 \cdot x^3$

$2 - 18 \cdot x^2$

ALG83 takes an expression (or allows the user to pick up whatever is in **Y1**), presents it correctly and then offers a choice of options: simplify (or expand), substitute a value, differentiate or view the function using table of values or graph. It also links directly with a second program for equation solving.

EQNSOLV takes the left-hand and right-hand sides of an equation (or uses **Y1** and **Y2** if desired) and offers the choice of a “fast solve” (using the in-built **SOLVE** command, or a step-by-step linear equation solver.

In the example shown, the first step would be to **SIMPLIFY**, expanding the left-hand side. From the menu shown, choosing 2 produces a simpler form. The next step involves, perhaps, taking $3x$ from both sides, so the user enters $-3x$. This is automatically simplified to give the next step of the process. In this way, students may work through the equation solving process, supported by the software until they are capable of working unassisted.

Enter an action.
e.g. -2X or +3.
Type 1 for menu,
2 to simplify,
0 to finish.
?

4·x-2=3·x+5

(4·x-2)-3·x=(3·x+4)

x-2=5

So what algebra did you want to be able to do with your high school students: simplify, expand, substitute, solve equations, even some calculus? Providing such tools offers suitable scaffolding for students in the early stages of algebra, allowing them to build up confidence and skills before putting aside such aids and working unassisted. There will always remain some students for whom algebra remains a mystery, just as the multiplication tables elude others. The latter are not disadvantaged in later schooling and in life because they are given access to calculators. It makes sense that students should be similarly supported in having access to available tools for symbol manipulation, even simple Computer Algebra Systems (CAS) such as that described here.

Once again, the supporting files described here may be downloaded from:
<http://homepages.ihug.com.au/~arnolds/alg83.zip>.

LEARNING ACTIVITY A4: FROM MOTION TO MEANING

APPLICABLE APPS: TOPICS IN ALGEBRA, CBL/CBR™/ SIMCALC MATHWORLDS

The study of gradient offers a very clear link between the algebra of the junior secondary school and the calculus of later years. If introduced carefully, the groundwork for later studies is firmly laid and students will appreciate the rich network of links which are so important to mathematics. This approach makes full use of the various technological tools available for the TI-83 Plus – especially the excellent software apps which are now available.

All algebra should begin with number. Students need to study the properties and relationships between numbers in order to extend and generalize this understanding using the symbolic tools of algebra. Prior to beginning this topic, it is worthwhile playing some guess my rule games with the students, using tables of values. Using a TI-83 Plus ViewScreen™ panel, the teacher may enter a function with the OHP turned off, and then display the table of values for the students to work out the hidden rule. The “magic number” – the gap between the values which students intuitively look for – is the beginning of the concept of gradient for them. Once comfortable with this process, it is time to move to a more complete treatment.

The *Topics in Algebra (Part 1)* app for the TI-83 Plus provides a thorough and innovative coverage of many of the key concepts required for the study of algebra in the junior school. By using a combination of clear, step-by-step instruction, eye-catching animation and regular opportunities for feedback, this application makes use of fundamentally good teaching techniques to ensure both understanding of the essential concepts, while at the same time ensuring the skill development which is so important. Section 3 on *Linear Functions* carefully and clearly introduces the study of gradient as slope in an intuitive and interactive way, while ensuring that the relationship between vertical rise and horizontal run is also reinforced.

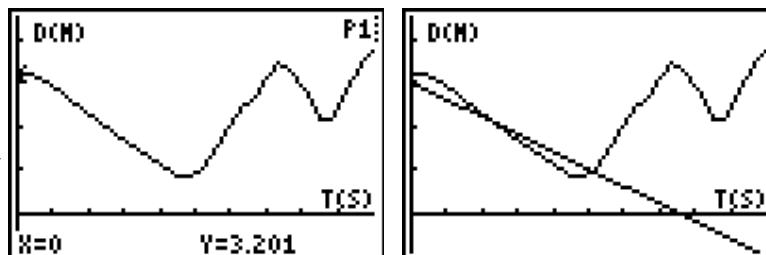
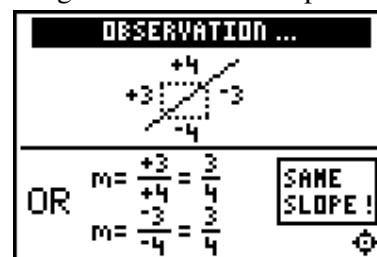
At the same time, there can be no better way for students to learn their mathematics than through physical interaction and the use of the Calculator-Based Ranger™ (CBR™) motion detector should be a part of every mathematics classroom. Having begun the formal study of gradient, it is now time for the students to experience it for themselves!

After some informal activities in which they become familiar with the functioning of the device in a fun and active way, it is time to move to a more rigorous treatment. This involves

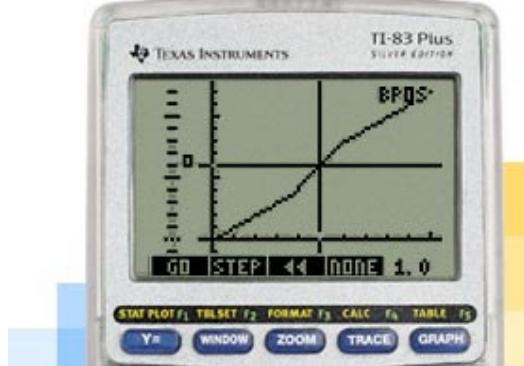
having students (usually in groups) using trial-and-error methods to fit straight line graphs

X	Y ₁
0	3
1	5
2	7
3	9
4	11
5	13
6	15

X=0



onto the motion graph which they have created. Quickly they come to realize, both the importance of rise and run, but the way in which these relate naturally to the speed and direction of their motion. This is a clear opportunity for teachers to build bridges to understanding which will lead them forward when they later come to the study of rates of change and differential calculus.



But wait! There is more. One of the more exciting concept apps developed in the past twelve months is called *MathWorlds* and has been developed as part of the ongoing *SimCalc* project, which seeks to develop tools which allow students to learn much of their mathematics in the context of real activities. The basic premise of this project is that mathematics should always be *about something*. All too often, it is presented to students in a formal decontextualised manner which not only defies

their understanding, but bores them silly. *MathWorlds* offers powerful animation tools by which students may observe and investigate real-time motion, and then create their own motions – both directly onto the calculator AND using a CBR™ to introduce their own physical movement to the scene. This tool offers powerful and significant mathematical opportunities to students from an early age to begin building concepts which will form the basis for their later study of calculus.

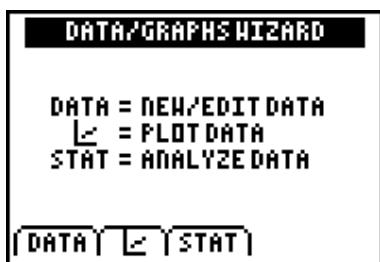
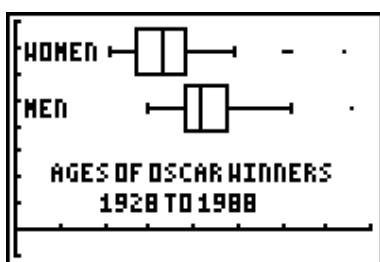
At the end, of course, understanding is not enough. Our students must also build strong numerical and algebraic skills which give voice to their understanding. In the study of gradient, I have always stressed three little words to help my students to calculate slopes given ordered pairs: *how far from...?* Every subtraction is a question which asks *How far from?* When subtracting 2 from 5, we are, in fact, asking “How far from 2 to 5?” This simple approach makes the calculation of gradient very easy for students, if they remember that gradient involves “Rise over run” (or, more formally, *y over x*). Thus to find the gradient of the line between the points (-2, 3) and (4, -1): begin with the two *y*-numbers and ask: “How far from 3 to -1?” With a little practice, they can confidently answer -4 (since to go from 3 to -1 we go *down* 4 steps!) Then, “How far from -2 to 4?” Clearly this is up 6 steps. The gradient, then, is given by the fraction -4/6, or -2/3. What could be simpler?

3. DATA

Overview: Data Representation, analysis and evaluation

The development of skills and understandings related to the gathering, organization and analysis of data features prominently across all stages. For many people, the principal way in which they encounter mathematics in their daily lives springs from access to quantitative information (especially that presented in tables and graphs), and effective functioning in today's world relies strongly on the ability of individuals to make sense of such information.

CALCULATOR APPLICATION AND APPLICABLE APP: *SciTools*



There can be little doubt that one of the great strengths of graphic calculators lies in the ease with which they support the entry, display and statistical analysis of quantitative data. Even for quite young students, data may be easily entered using the list editor; with assistance, this may be displayed in a variety of graphical forms.

This process may be made even easier by use of the *SciTools* app, which offers a “Data/Graphs Wizard” specifically designed to simplify the process for those who may not be familiar with all of the calculator’s functions. As shown, choosing between **DATA**, **PLOT** and **STAT** allows the user to easily enter (or edit) data into lists, to choose from 4 different graph types and even to calculate the statistics for the lists entered – all without needing to refer to the menu functions of the calculator itself.

In this way, the statistical features of the calculator are made available to a much wider audience.

L1	L2	L3	2
22	44		-----
36	38		
26	46		
62	53		
32	35		
26	47		
29	34		

L2(1)=44



1-VAR STATS FOR L2	
MinX	= 30
Q1	= 38
Med	= 42
Q3	= 48
MaxX	= 76

LEARNING ACTIVITY D1: MAKING PEOPLE GRAPHS

Students learn best when they are physically involved. When preparing any lesson, the key question for teachers should not be “*What am I going to do to teach this concept or skill?*” but rather “*What are THEY going to do to learn this concept or skill?*” Physical involvement (when well managed) guarantees a lesson that is likely to be memorable for students. It is the task of the teacher to ensure that at least part of what they remember is the mathematics that lies at the heart of the activity – not just the activity itself. Students are also readily motivated by lessons which refer to themselves – in this case, birthdays.

The various types of statistical graphs may be readily modeled by students themselves. Consider the following.

1. Pie Chart

For this activity, we might use the *season* of their birth. Explain to the class that we are going to line up in order of the season of our birth. Ask them to think about and discuss what they expect to happen here: is there likely to be any particular pattern in this, or would we expect people to be evenly spread among the four seasons?

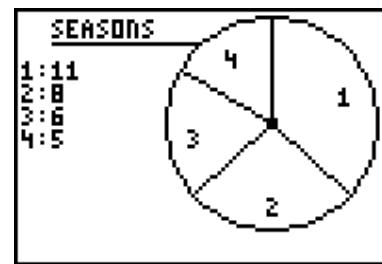
Call out all the Spring babies – those born in September, October or November. Have them form a line (for a class of 30, this may need to be done outside, since it begins with one long line). Note that order is not important here.

Next call out the Summer babies – December, January, February. Follow with Autumn (March, April, May) and, finally, Winter (June, July, August).

Have students look along the line and comment if any pattern is visible. You may want each season to step forward.

Finally, while remaining in their order, have the students form a circle and sit down, equally spaced. Now they have formed a human pie chart. In turn, each season should stand, or hold up their hands for everyone to see. Is it easier now to see how the children are distributed through the seasons? Is the result what was expected?

Discuss the 360 degrees of the circle and how each student is related to this: if there are 30 students in the class, this is not hard. If not, they may use their calculators to calculate how many degrees each student represents.

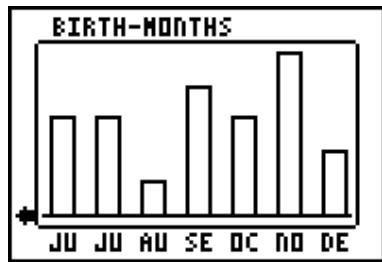


On returning to class, students should draw up the pie chart they have just created. If they have the *CellSheet™* App, they should enter their numbers and compare their pie chart with that generated by the calculator. Find the percentage of the class born in each month, and ask them to comment on this result in their workbook.

2. Histogram

For this graph, you may use their seasons, or it may be more interesting to use their month of birth. Describe the activity, and then have students record any predictions they may expect in their journal or workbook. Talk in class for a few minutes about these expectations: do they expect that they will be equally spread among the months? Why, or why not?

Once again, call students out by month of birth, and have them line up in parallel rows, equally spaced (they may space themselves by putting one hand on the shoulder of the student in front, and one hand to touch the student at their left or right. Once again, this is probably best done outside with plenty of room.



Have students sit down, and then have each month stand up and observe the human histogram they have formed. Record the data in the calculator's list editor (Press **STAT**). Enter the numbers from 1 to 12 in **L1**, then the numbers in each month in **L2**.

Before returning to class to write up this activity and to draw their histogram, have the last student in each column reach across to the last person in the next column, forming a line graph with their arms. Have students talk about this, and why the line graph would NOT be appropriate in this case. Ask them to think about situations they could model where a line graph would be better than a histogram.

Why was a histogram a better choice for this data than a pie chart? Why was a pie chart the better choice for the first activity?

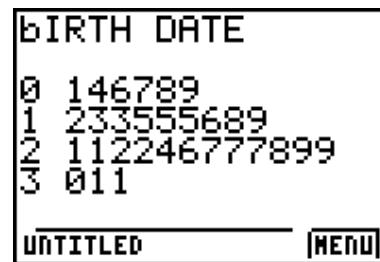
3. Stem and Leaf plots

For this activity, we will use the *day* of their birthday: i.e. if they were born on the first or the 21st of a month. The class will be divided into numbers between 1 and 31, ignoring the actual month of their birth: we are only interested in the day of the month. Again, have the class discuss what they expect to see in this graph: are they likely to be equally spread through the 31 days available? Why, or why not?

Call out all those with single digit days: line them up in increasing order from a starting point: call them 01, 02, 03, etc. Ask them to sit down.

Now call out those born on a day between 10 and 19: line up in order parallel to the first group, and sit down. Line up the 20s next, sit them down, and then the 30s.

They have formed a stem and leaf plot. Each line should stand up in turn and study the graph, noting any features of interest.



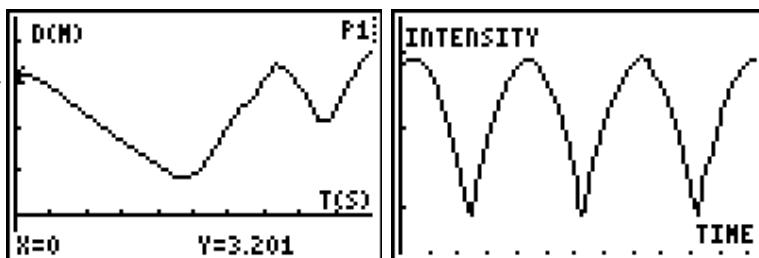
Finally, ask the boys to separate from the girls: girls on one side of the middle line, boys in order on the other to form a back-to-back stem and leaf plot. Look, think, talk.

Go back to class and write this up: draw both stem and leaf plots. Enter the data into lists as before and look at the graphs formed. What is the advantage of a stem-and-leaf plot over a histogram?

Learning Activity D2: Gathering Real-World Data

APPLICABLE APP: *CBL/CBR*TM

Data comes in many forms and students should encounter a great variety of statistical information in their studies. It may be generated by class activities and experiments, by personal and family information, from Internet research or even from newspapers and books!



One particularly powerful and appropriate way in which data may be generated involves the use of data collection devices (data loggers) which work in conjunction with graphic calculators. In this way, real-world data of all kinds may be collected, viewed and analysed, providing students with a wealth of skills and experiences across a wide range of applications. The first screen-shot shown displays the result of a student moving backwards and forwards in a classroom, as captured by a sonic motion detector (*CBR*TM: *Calculator-based Ranger*TM). Such a device may be used in many ways: one example involved the Ranger being attached to the top of the classroom door-frame; as the class entered the room, their heights were measured as they passed under the detector, allowing further examination of this data. This offered a quick, easy and fun way to collect and display such information.

Later in the high school years, such graphs may readily be linked to a variety of mathematical functions, from linear to trigonometric, and more. The second screen-shot displays the result using a light probe to measure light intensity generated by a fluorescent light. The rapid oscillation of this type of light (usually invisible to the human eye) is clearly demonstrated in the shape of a trigonometric function. When a data logger with a range of probes (beginning with simple ones, such as light, temperature – even a heart-rate monitor!) is used, such modelling activities become powerful mathematical learning experiences for students of all ages. They also provide the basis for much useful discussion and perhaps the odd argument!

LEARNING ACTIVITY D3: ORGANISING AND DISPLAYING REAL-WORLD DATA

APPLICABLE APP: *CELLSHEET*TM

WEIG	B	C	D
1	PLANET	GRAY	WEIGH
2	M	.38	28.5
3	V	.91	68.25
4	E	1	75
5	M	.38	28.5
6	J	2.54	190.5

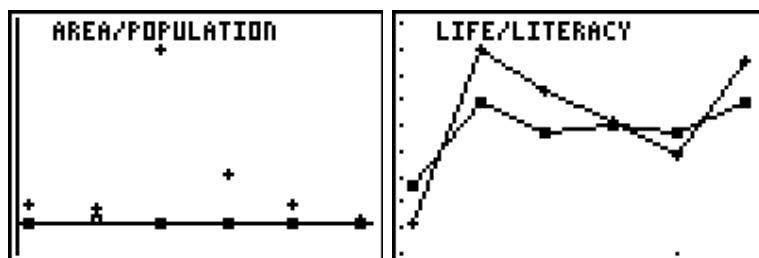
D2: =\\$E\\$1+C2 [Menu]

The inclusion of a full-featured spreadsheet on a graphic calculator significantly extends the capabilities of this already powerful device. Like the word processor, the spreadsheet has become an indispensable technological tool for both business and school. Students from the upper primary years are expected to be familiar with at least the basic operation of this application, and the *CellSheet*TM app provides a consistent and compatible version for students and their teachers to use in any classroom.

While the list capabilities of the graphic calculator are flexible and powerful, the versatility of the spreadsheet makes its use attractive. As demonstrated above, text may be mixed with quantitative information, and the whole linked interactively. In the example shown, the relative gravities of the nine planets in the solar system are used to compare my weight on each (assuming Earth's gravity to be 1, and my weight to be 75 kilograms). At a glance, it can be seen that my weight on both Mercury and Mars is about the same (28.5 kg), while my weight on Jupiter suggests that this will not a desirable holiday destination for the wealthy in the future! By simply changing my weight, however, the whole sheet automatically updates and allows for others to quickly and easily investigate the effects of gravity on other values.

DATI	B	C	D
1	AREA	POPUL	
2	AFGHAN	652000	2.58E7
3	AUSTRA	7.69E6	1.92E7
4	INDONE	1.92E6	2.25E8
5	IRAN	1.65E6	6.56E7
6	IRAQ	337072	2.27E7

B1: [Menu]



The spreadsheet allows the organizing, display and (if desired) analysis of many different types of data. Consider the example shown, in which information drawn from the *World Geography* app is collected and entered for a range of nations: area, population, life expectancy, literacy and Gross domestic Product. The essential task of all statistics is how to make sense of large amounts of information. Sometimes, this involves organizing this data graphically, giving a visual overview of properties and trends. Sometimes, the task involves reducing the data to a small number of pieces of information which capture the essence of the original data in some way. Hence, measures of location (mean, median, mode) and measures of spread (range, deviation) reduce potentially huge amounts of information down to a few numbers. Much is lost in this process, but often much is gained.

3.

MEASUREMENT

Overview: Measurement

Across the stages, the Measurement Strand allows for a gradual development in complexity of both skills and understandings as applied to this very practical component of the course. The introduction of Pythagoras' theorem is a pivotal development in early high school, extending the range of measurement applications to a much broader field. Conversion and appropriate choice of units, use of intuitive and formal methods for calculating areas, perimeters and volumes and a growing awareness of the issues associated with accuracy and precision in measurement are all features of this strand.

LEARNING ACTIVITY M1: CONVERTING UNITS

APPLICABLE APP: *SCI TOOLS*

The image shows two screenshots of the SciTools app. The top screenshot is a 'UNIT CONVERTER' menu with categories: LENGTH (1), MASS (7), AREA (2), FORCE/WT (8), VOLUME (3), PRESSURE (9), TIME (4), ENERGY/WORK (A), TEMP (5), POWER (B), and VELOCITY (6). It also includes 'SI PREFIXES' (C) and '(CONSTANT)'. The bottom screenshot is a 'LENGTH' conversion table with columns for fm, m, mm, cm, and in. It shows conversions between fm and m, m and in, and m and ft. Below the table are two results: 2.16E4 nmile and 4.00032E4 km. At the bottom are buttons for '(CONSTANT)', '(EXPT)', '(COPY)', and '(EDIT)'.

Just as technology may serve a scaffolding role with regard to number and algebra, so it may be used to support the learning of measurement. Conversion between different units of measurement is a common source of error for students and one which requires practice over time. The *SciTools* app offers a convenient unit converter which may be used for very simple conversions (from 2500 mm to 250 centimeters) to quite difficult (from 21600 Nautical miles to kilometers – approximately 40 000 km, the circumference of the earth at the equator). While the automatic conversion of entries into scientific notation will challenge some students, there are decided advantages in building up some familiarity with this notation along with the introduction of index notation in the number strand.

This App offers a powerful and useful resource for students in building up their understanding of the relationships between the different metric units, and offers possible extension work for those interested to go beyond the standard treatment of this work.

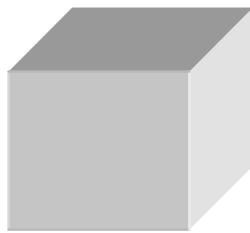
AIM: *To build student understanding of the relationship between volume and capacity.*

METHOD: *This simple and fun activity invites students to use their imaginations and unit conversion skills to solve what appears to be a simple problem with a surprising result.*

PROCESS:

1. Begin with a box 1 metre on each side. This may be a real box if you have one, or an imaginary one that you trace out for the class. Use a metre rule to establish the dimensions and to clarify students' image.
2. Show students a 1 litre container (a milk carton works well).
3. The problem: *How many trips must I make to the tap with my 1 litre carton in order to completely fill the box?*
4. Provide one further piece of information: *1 cm³ holds 1 ml of water and weighs 1 gram.*

The answer: 1 000 trips! And, when full, the box will weigh 1 tonne!



LEARNING ACTIVITY M2: COVERING AREAS

APPLICABLE APP: AREA FORMULAS

The *Area Formulas* App is an example of a well-defined teaching sequence which may be used from as early as the upper Primary through to the Senior Years. It carefully and thoroughly defines the six common shapes shown, demonstrates (in some cases using animation) the area formula for each shape, and offers a variety of examples. The **WHY?** Menu offers an illustrative explanation for the rule, which makes it clear to students. Finally,

students may take a quiz and consolidate all that they have learned.

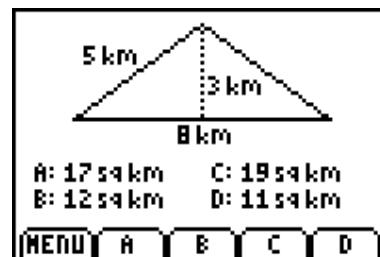
This App was clearly written by teachers for learning. While it does include some non-metric units of measurement, these do not detract from the study of the formulas required. Students may work through this app individually, in small groups or as a whole class using the ViewScreen™. Naturally, it provides a starting point for the study of volumes and surface areas as well.

AIM: *To use the study of area as an opportunity for review of number skills and the introduction of irrational numbers.*

METHOD: *Heron's Formula defines the area of a triangle in terms of the lengths of the three sides, using the semi-perimeter S (half the perimeter of the triangle). If A , B and C are the sides of a triangle and $S = (A + B + C)/2$, then*

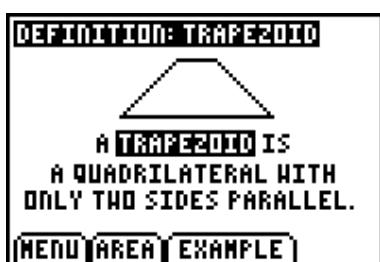
$$\text{Area} = \sqrt{S(S - A)(S - B)(S - C)}$$

PROCESS: *Use a triangle such as the one given to test this formula.
 Make some of your own.
 Will it always work?*



LEARNING ACTIVITY M3: TRAPS WITH TRAPEZOIDS

APPLICABLE APP: AREA FORMULAS



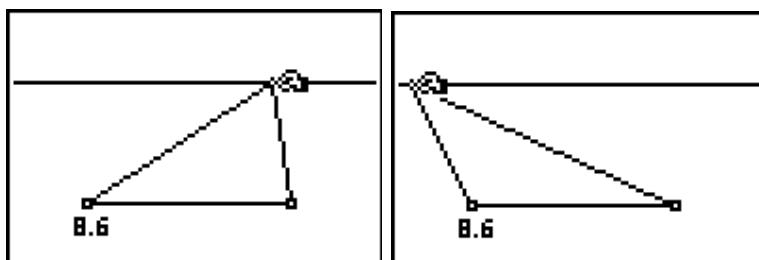
The NSW K-10 Mathematics syllabus defines a trapezium as a quadrilateral with *at least one* pair of parallel sides.

How does this differ from the definition provided in the *Area Formulas* App?

Is this an important difference? Why or why not?

LEARNING ACTIVITY M4: DRAGGING SHAPES AROUND

APPLICABLE APP: CABRIJR™

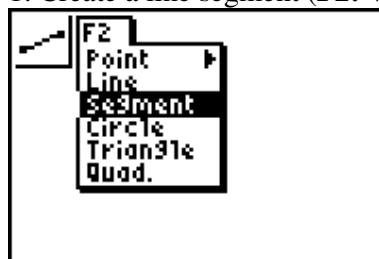


The true power of interactive geometry software (such as the new *CabriJr™* App) lies, not only in the ability to transform, but in seeing what remains the same under such changes.

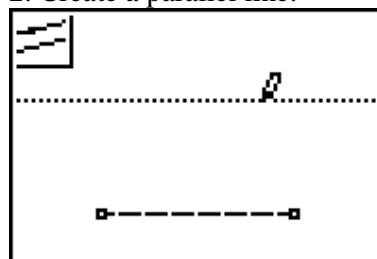
AIM: To build student understanding of the relationship between the area of a triangle and the base and perpendicular height.

METHOD: By constructing a triangle between two parallel lines, students are free to transform the shape by dragging a point along one of the given lines. The area under these conditions remains invariant.

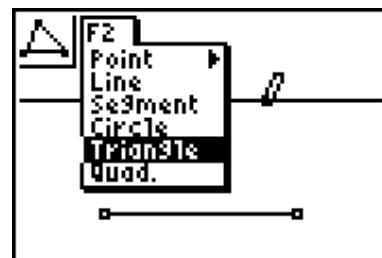
PROCESS: 1. Create a line segment (F2: Window).



2. Create a parallel line.

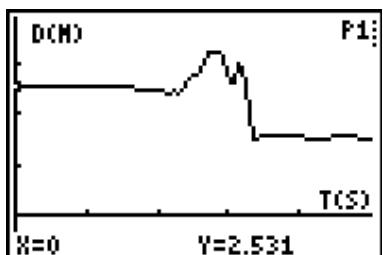


3. Create a triangle using a point on the line and the endpoints of the segment.
4. Choose F5: Measure: Area to find the area of the triangle as shown above.
5. Discuss why the area remains the same – what is not changing?



LEARNING ACTIVITY M5: HOW FAR AM I?

APPLICABLE APP: **CBL™/CBR™**



While the Calculator-Based Ranger™ (CBR™) may be called a “motion detector” it can prove just as useful with non-moving things. In fact, this handy little device is one of the quickest and easiest tools for measuring distances and lengths. Because students are physically involved in the activity they will remember it well. They are also encouraged to use a “guess and check” process to refine their estimates. This builds a willingness to “have a go” whether they are sure of the answer or not.

AIM: *To assist students in building skills of estimation for distances in metres.*

METHOD: *Students are invited to make guesses of given distances and to actually move to a position which demonstrates their guess. The CBR™ can then quickly measure and verify this distance.*

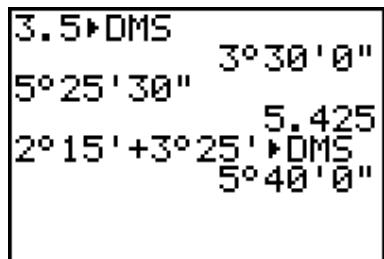
PROCESS:

1. Clear a space in the classroom or move outside and choose a suitable location. Ideally, find a place where there is a straight line of some sort from which students can take their measures.
2. Sit the class around the chosen area and choose the first few volunteers (3 – 5 students is normally a good start).
3. Give them a distance to estimate, say 3 metres. They take a position this distance from the line. You may have them do this one at a time.
4. While students remain still in their positions, another student with a CBR™ walks along the line, measuring the distance of each volunteer.

See who was the best guesser, then repeat with other students.

Try variations such as trying to position yourself your own height from the line!

LEARNING ACTIVITY M6: WHAT TIME WAS THAT? CALCULATOR APPLICATION



Time conversions can be difficult, and yet they are among the most common measurement units that students will encounter, even from a very young age. The calculator's ability to work with degrees, minutes and seconds (angle measurements) offers a fortunate overlap with the measures of time. Instead of "degrees", simply substitute "hours" while minutes and seconds remain the same.

Consider the three conversions shown above.

1. To convert 3.5 hours into hours and minutes: Enter 3.5, then choose the **ANGLE** menu (2nd **APPS**) and option 4, which converts to degrees, minutes and seconds (or, in this case, 3 hours and 30 minutes).
2. 5 hours 25 minutes and 30 seconds is entered, again using the **ANGLE** menu: choose option 1 for degrees (or hours), option 2 for minutes. To choose seconds use **ALPHA** +, since the seconds symbol is in green above the + key.
3. *My flight was in two legs: the first, two hours and 15 minutes, the second took 3 hours and 25 minutes. For how long did I travel overall?*

LEARNING ACTIVITY M7: LINES OF TIME APPLICABLE APP: *TIMESPAN*



TimeSpan is a new concept App which supports the creation of multiple annotated timelines. Whether students are learning about the lives of mathematicians, the ages of dinosaurs or the growth and development of mathematics, the applications of this tool are limited only by our own imaginations. Students may use *TimeSpan* to organize and present the results of research on some mathematical topic, or to learn more effectively about number lines, ordering of numbers and intervals or any of a wide range of learning situations.

LEARNING ACTIVITY M8: ROUND ABOUT PYTHAGORAS CALCULATOR APPLICATION

This activity forms a suitable investigation after studying Pythagoras' Theorem, serving to reinforce the knowledge and skills, while pointing to more interesting things to come.

Students begin by drawing a 10 cm line. They are then asked to construct a right-angled triangle with this line as the hypotenuse. They may, if they wish, work in pairs. Many will produce a 6-8-10 triangle.

They are then challenged to find as many different right-angled triangles as they can, with a 10 cm line as hypotenuse. They may, if they wish, use aids such as a set square, or any right-angle they can find (the edge of a book, a piece of paper, the corner of a ruler). If students are finding this task too difficult, the teacher may, after a while, suggest that they cut a line of 10 cm length in a sheet of paper, then poke another sheet through that hole as far as it will go, corner first. This forms the required triangle.

When they think they have a suitable triangle, students should carefully measure the other two sides, and then use Pythagoras' Theorem to check that these lengths are suitable – the result of adding the squares of these numbers should be close to 100 (Why?).

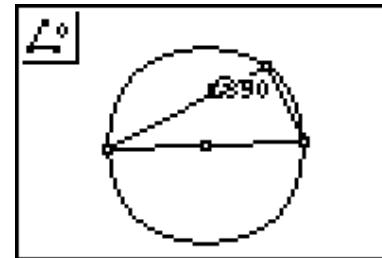
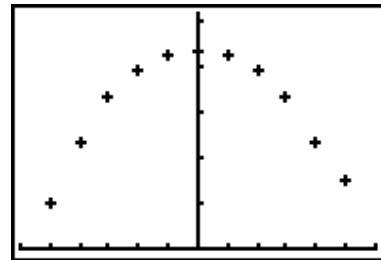
Once satisfied, they may come to the front, and enter their two numbers into **L1** and **L2** on the teacher's calculator, connected to a ViewScreen™: the first number in **L1**; the second in **L2**. If desired, you may instruct them to then enter the numbers a second time in reverse order – since every pair works for *two* triangles. Discuss this.

When all the class has entered some pairs, the resulting scatter plot should reveal to the students the beginnings of a semi-circle. Is this what they expected? Discuss what this means.

They may carry the investigation further by actually cutting out each of their different right-angled triangles, and laying them one on top of the other on the overhead projector, all sharing the same 10 cm hypotenuse. The tips of the triangles again should show the shape of the semi-circle.

Finally, some students may wish to investigate further, and there is no better tool for this than a interactive geometry package such as *CabriJr*™ or *GeoMaster*™.

Whatever the tools of investigation, ensure that all students reflect upon and comment upon the *mathematics* of what they are doing. Invite them to put their thoughts into words – first with a partner, then more generally. And always finish by having them write about their experiences and observations.



5. SPACE AND GEOMETRY

Overview: More than Shapes and Position

At the start of high school, it is expected that students will be familiar with common two- and three-dimensional shapes and objects, and their features and properties. The beginning of high school represents a significant shift in the nature of this study: geometrical properties of angles and shapes are organized logically and used to develop further complex understandings related especially to triangles, quadrilaterals and circles.

LEARNING ACTIVITY G1: GETTING STARTED WITH INTERACTIVE GEOMETRY

Begin with a point.

What could be simpler or more enticing?

Welcome to the elegant and beautiful world of interactive geometry, in which the static images of the past make way for strange flexible creations which offer insight into many of the most fascinating parts of mathematics.

Be warned, however. As teachers, it lies within our power to make the extraordinary appear mundane.

Some approach the blank screen of interactive geometry with a recipe book of step-by-step instructions, ensuring that they “cover the work required”. This will most certainly guarantee boredom and rapid dissatisfaction on the parts of both students and teacher.

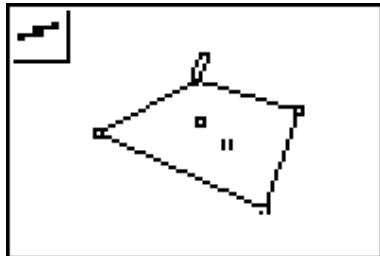
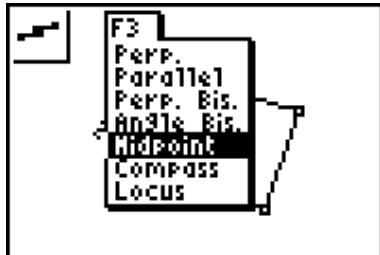
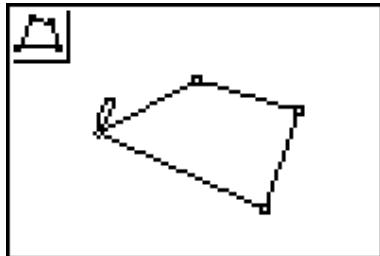
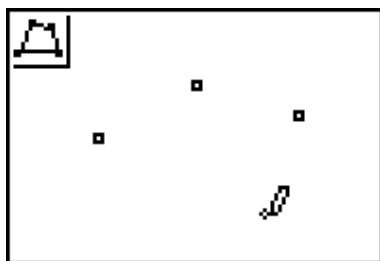
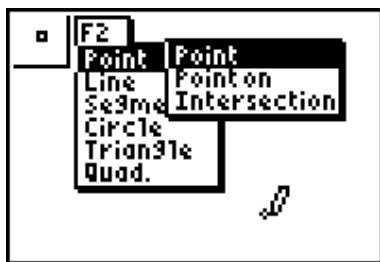
The secret to effective use of technology in the classroom has always been the same.
Learning to use the new tools is about learning to ask new questions.

This is the challenge of open-ended tools. They make trivial much that we previously devoted great amounts of classroom time to. Too often, “...*the first instinct of educators is to couple the technology to their old methods of instruction.*” (Papert, S. [1980] *Mindstorms: Children, computers and powerful ideas*. Brighton, UK: Harvester Press). Like Seymour Papert, our “*vision should be of something much grander*”.

The few simple activities which follow may offer some guidance in this new approach to the teaching of geometry. The questions being asked are simple yet rich. They invite students to engage with important ideas and the role of the teacher shifts from being the source of knowledge to being co-learner. Our primary responsibility, however, remains unchanged. We need to ensure that students take from the learning experience *at least* what we aimed for them to achieve. Hence we need to have them *reflect* upon what they discover, *record* it in their own words, make meaning of their experience, and *compare with others* in order to deepen and validate their new understandings.

LEARNING ACTIVITY G1: MIDDLE OF THE ROAD

APPLICABLE APP: CABRIJR™



1. Begin with a point. Choose **POINT** from the drop-down **F2 (WINDOW)** menu and simply click wherever you wish to place a point. Now add three more.
2. Choose **QUAD** from the same menu and then click once on each of the points you have created. You may save time by clicking twice on the third point and the quadrilateral closes off automatically.
3. Press the green **ALPHA** key when close to one of the points and watch the pencil change into a hand, allowing you to drag this point (and the rest of the quadrilateral) around. You have not created a quadrilateral, you have created many possible quadrilaterals!
4. Press **F3 (ZOOM)** and choose **MIDPOINT**. Click once on each of the *opposite* corners (vertex points) of the quadrilateral. This constructs the midpoint of each of the two diagonals.
5. Now again choose a point, press **ALPHA**, and drag it around trying to make these two midpoints coincide.
6. *What shape have you formed? Check with others: did everyone produce the same shape? Try this again using another quadrilateral until you are sure about your result.*
7. Write down what you have observed and what this tells you about quadrilaterals.
8. A *theorem* is a statement in mathematics that may be true, but needs to be proved. Work with a partner or a small group to write down a theorem that you have discovered from this activity. How might such a statement be *proved to be always true!*

LEARNING ACTIVITY G2: HOW SQUARE!

APPLICABLE APP: *GEOMASTER*™

In how many ways can you construct a square?

AIM: *To encourage students to explore the various menus and features of their interactive geometry package while investigating properties of a square.*

METHOD: *After checking that students all know what a square is, give them the question above and let them loose! You may choose to offer a reward for the student or group with the most different methods.*

PROCESS: Most students will begin with a segment, representing the first side of the square.

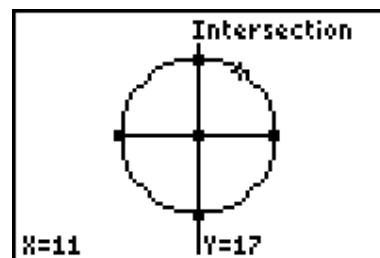
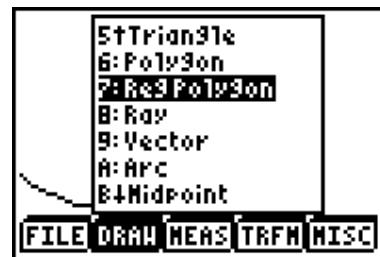
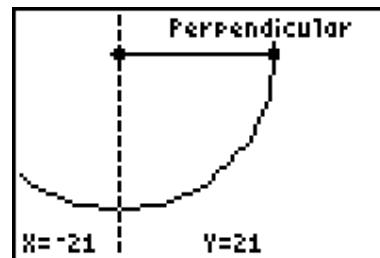
Some methods may include constructing a perpendicular line through one end point, then using a circle to mark the position on this perpendicular, which corresponds to the length of the first segment.

Alternatively, try rotating a copy of the first segment around one end through 90 degrees.

If you are using *GeoMaster*™, why not go straight to the **Regular Polygon** command on the **DRAW** menu? (It begins by default with 6 sides, but pressing + and – changes the number of sides).

A special prize should go to anyone who thinks in terms of diagonals! Begin with the diameter of a circle. Create another equal diameter perpendicular to the first. The four intersection points with the circle form a square!

Any more?



LEARNING ACTIVITY G3: WHAT IS YOUR ANGLE?

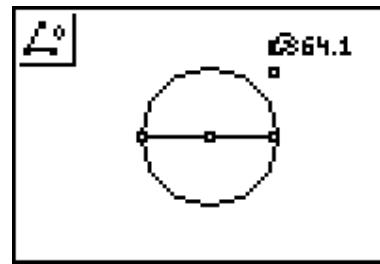
APPLICABLE APP: **CABRIJR™**

AIM: *To develop students' understanding of angles both acute and obtuse by introducing an important circle property to which they will return later.*

METHOD: *This simple activity not only introduces Thales' Theorem (the angle on the circumference of a semi-circle is always a right angle) it breaks students from the usual perception of angle by deliberately not constructing the arms.*

PROCESS: Begin once again with a point. Anywhere. Then construct a segment (independent of the original point), put in the midpoint and use this as the center of a circle with the segment as diameter.

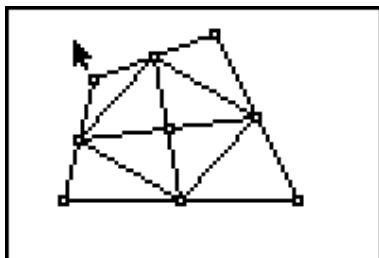
Finally, measure the angle from one end point of the diameter, to the free point, then to the other diameter end point.



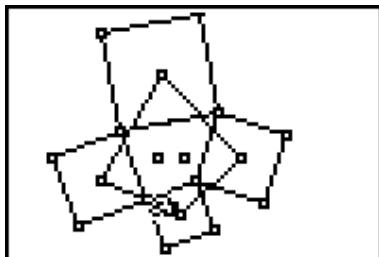
You may now begin to move the point around the screen, observing the way that the angle changes in relation to the position of the circle. Record and discuss observations.

Later, you may choose to allow students to create the triangle defined by the three points and make explicit the nature of the angle. Some students will be quite uncomfortable until this is done. Talk about angles.

SOME OTHER INVESTIGATIONS



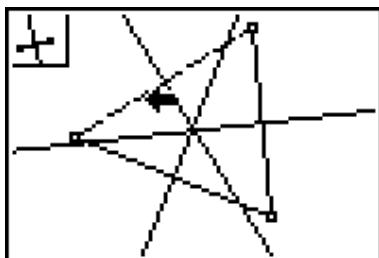
1. Begin with a quadrilateral – any quadrilateral. Construct the midpoint of each of the four sides and join these as shown.
What may be observed about the new quadrilateral formed.
Will this always be true?



2. Once more, begin with a quadrilateral. Construct a square on each side as shown. Find the midpoint of each square and join these four points. What may be observed about the quadrilateral formed?
Will this always be true?

What would happen if regular shapes other than squares were used (such as equilateral triangles)?

Did you know that the famous general, Napoleon Bonaparte, was an amateur mathematician. He is remembered for his work related to a simpler version of this figure (with equilateral triangles on each side of any triangle) and the general result is known as Napoleon's Theorem.



3. We all know about the center of a circle. *But how many centers has a triangle?*
Join each vertex of any triangle to the midpoint of the opposite side. These three segments are called medians and each meets in a special point called the *centroid*. This point is the *center of gravity* for the triangle: you should be able to balance the triangle on a pin at this point. Cut a triangle out of cardboard and try this.

Taking the perpendicular bisector of each side (as shown) yields three lines which all meet in a single point, called the *circumcentre*. Try to find out more about this special point.

See what other centers you can discover and then research these to find out more about them.

LEARNING ACTIVITY G4: WHAT IS YOUR ANGLE?

APPLICABLE APP: *WHAT'S MY ANGLE?*



This superb app provides students (and their teachers) with the opportunity to build up skills of angle estimation, while reinforcing the properties of acute, obtuse and reflex angles. It is great fun to play with the entire class, individually or in small groups. The animation and graphics are of good quality and students are mesmerized by the activities.

My experience with groups of both teachers and students is that most people tend to be over-estimators when it comes to angle: even those of us with many years in the business! As usual, scores may be kept and improvements recorded if desired. This one has entertained and educated students in my classes from upper primary to the senior years!

LEARNING ACTIVITY G5: WHERE IN THE WORLD?

APPLICABLE APP: *WORLD GEOGRAPHY*

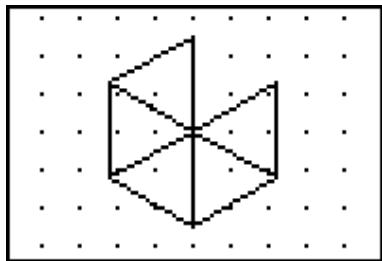


This amazing database of information has applications in just about every subject, and mathematics is no exception. From early mapping skills to the building up of concepts

associated with latitude and longitude in relation to the earth as a three-dimensional globe, there is much here that is worth spending some time on. Applications are not limited to space and geometry, of course: areas, large numbers, proportions, currencies and many more pieces of information will provide great learning opportunities in just about every strand. Students in later years will use this information to calculate time differences between world cities, but there is nothing to stop teachers introducing key ideas whenever they feel it is appropriate – and access to this tool makes that even easier.

LEARNING ACTIVITY G6: TURTLING AROUND

APPLICABLE APP: *LOGO*



direction.

We all know what a square is, don't we?
But what if you are a robot, who only knows four
commands: forward, back, left and right?
What instructions would produce a square?
Having one student serve as the robot and others giving the
instructions proves a powerful and effective learning
mechanism, which produces surprisingly robust
understandings related to distances, angles, position and

The original *LOGO* computer language was designed by Seymour Papert and his team at the Massachusetts Institute of Technology over thirty years ago. It was designed around the motto “No floor, no ceiling” with its implication of accessibility for all, but the potential for learning experiences appropriate for any age. It remains a powerful environment for learning mathematics, particularly concepts and skills associated with space and geometry.

The *LOGO* app offers basic “turtle graphics” (the turtle is the nickname for the triangular “robot” which draws as it carries out instructions). In the case of a square, students quickly arrive at the series of commands: FD 20 (or whatever distance is appropriate), LT 90, FD 20, LT 90, FD 20, LT 90, FD 20, LT 90 (again, a right turn would have been equally correct). This may be shortened to **REPEAT 4 [FD 20 LT 90]**.

This provides the starting point. Constructing a triangle always proves more difficult than expected (since the turn for an equilateral triangle is 120 degrees rather than the expected 60) but students begin to *feel* angles since they are physically involved in the process. This leads naturally to improved skills and concepts related to position and direction, and links directly with map reading and orienteering activities.

Other applications include tessellations. The pattern above was created using the command sequence: **REPEAT 5 [REPEAT 3 [FD 20 LT 120] LT 60]**.

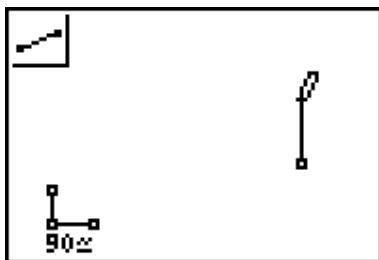
Every student should have the opportunity to work within the *LOGO* environment. In the words of its creator, it seeks to be a “mathland”, where learning the language of mathematics is as natural as learning French if you grow up in France!

LEARNING ACTIVITY G7: ALL SHAPES AND SIZES

APPLICABLE APP: CABRIJR™

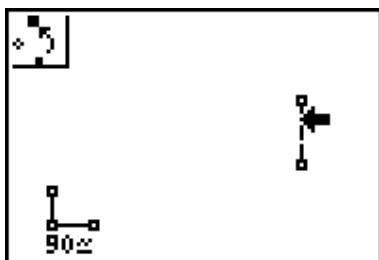


Once again, begin with a point. Place it in the bottom left corner of the screen, and construct two short segments coming out from this point as shown. The angle they make is not important at first, but we will now measure it.

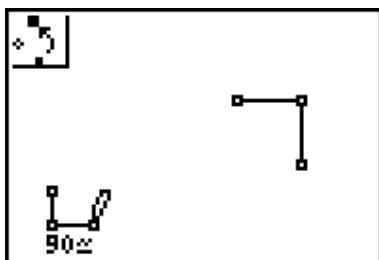


To measure this angle we need to click in turn on each of the three points which define it. Make sure that the central point (your original point) is the second point that you click on. So, choose **F5 (GRAPH): Measure: Angle**. Click on the last point you created, then the middle (or vertex point) then the third point. Place the angle somewhere out of the way, and then press **ENTER** to leave it there.

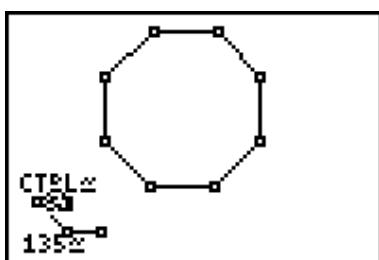
Now move to the other side of the screen and create another segment as shown. We are going to *rotate* this segment using the angle we have just created as the angle of rotation.



Choose **F4 (TRACE): Rotation**, then move down a little in order to select the segment you have just created (as shown). Move up to click on the end point of the segment – this will be the center of rotation. Finally, move across and again click on the three points making up our angle (do this in the same order that you just used to measure the angle). A new segment will appear, rotated by the number of degrees that you have chosen.



Now choose **F4: Rotation** again, select the *new* segment, and rotate it about its free end point by the same angle. Repeat this until you have closed your figure. Keep creating sides, or press **CLEAR** to leave Rotation mode.



Move to the top point of your angle, press **ALPHA** to grab it and move it carefully to the right, until the angle reads **60** degrees, or close to it. What do you observe? Now move to the left until the angle is close to **120** degrees. You may be able to adjust this angle more accurately by changing the length of the angle arm slightly (moving up and down). Try moving until the angle is about **150** degrees, then continue to rotate the last side until you form a closed figure once more. *Investigate the many regular polygons you have created!*

6. WORKING MATHEMATICALLY

Questioning

Students ask questions in relation to mathematical situations and their mathematical experiences.

Applying Strategies

Students develop, select and use a range of strategies, including the selection and use of appropriate technology, to explore and solve problems.

Communicating

Students develop and use appropriate language and representations to formulate and express mathematical ideas.

Reasoning

Students develop and use processes for exploring relationships, checking solutions and giving reasons to support their conclusions.

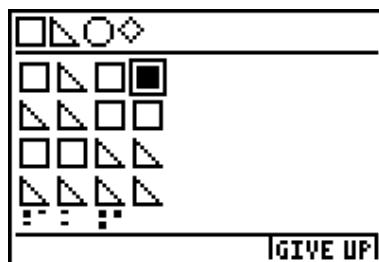
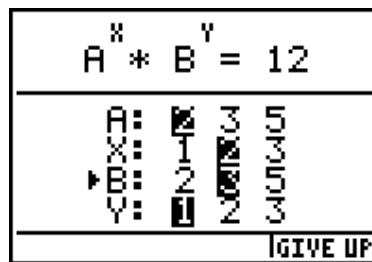
Reflecting

Students reflect on their experiences and critical understanding to make connections with, and generalizations about, existing knowledge and understanding.

Overarching all strands and experiences of the syllabus is the assumption that mathematics is not a body of knowledge but a process, a way of doing things and of looking at and making sense of the world. In the opening words of the previous Years 7 and 8 NSW Mathematics Syllabus, *Mathematics is a search for patterns and relationships*. Students are *doing* mathematics when they are looking for patterns, recognizing commonalities and actively organizing, questioning and communicating their experiences.

The premise of working mathematically permeates each of the apps described in these pages. Designed by teachers for teaching and learning, each offers a variety of mathematical experiences in such a way that students are supported and encouraged in the processes of questioning and communicating, reasoning and reflecting and learning to apply strategies. From the simple skill-based games of *Decimal Defender* and *What's My Angle* to the extraordinarily powerful blank page presented in *CabriJr* and *GeoMaster*, students are invited to enter a series of technological “mathworlds”, where the learning of mathematics becomes as natural as speaking, walking or riding a bike.

The final series of apps to be described here are best thought of as “problem solving tools” in that they appear not to fit neatly into any one of the strands previously described, and yet their nature and purpose is intimately linked with the goal of working mathematically.



1. LOGIC LADDER

Logic Ladder consists of two problem solving scenarios: the familiar (“classic”) MasterMind game (where the shape and

position of the puzzle pieces is worked out by a combination of guesswork and strategy) and a “factor” game, where students use their knowledge of factors to find the pieces to this puzzle.

Scores are kept and students have the opportunity to “give up” if it all gets too much for them.

2. SMILE MATHEMATICS

The games in this package offer a variety of skill development experiences, from ordering of whole numbers to place value and even equivalent fractions. This collection was designed by the Isaac Newton Centre for Mathematics Education in London.

3. SOLVE-IT!

This app really ensures that students understand their operations and the importance of that equals sign! In the example shown (Bronze level, first of three levels of difficulty!) the highlighted question is: *if A = 1, then what does B equal?*

You must then find the value for A when B = 41. Another excellent learning tool for students from Stage 4 onwards.

AVAILABILITY OF APPS

From <http://education.ti.com>

1. Topics in Algebra Part 1	\$US7
2. Area Formulas	FREE
3. CBL/CBR™	FREE
4. CabriJr	FREE
5. CelSheet	\$US15
6. Decimal Defender	FREE
7. GeoMaster	\$US15
8. Logic Ladder	FREE
9. SimCalc MathWorlds	Shareware version free
10. Math by Hand	FREE
11. Number Curiosities	FREE
12. ProbSim	FREE
13. Puzzle Pack	FREE
14. Rational Rampage	FREE
15. SciTools	FREE
16. SolveIt	FREE
17. StudyCards	FREE
18. TimeSpan	FREE
19. What's my Angle?	FREE
20. World Geography	FREE

From other developers (Search at <http://www.ticalc.org>):

21. LOGO
22. PrettyPrint
23. Symbolic
24. SMILEMathematics

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