TI-nspire™

The Art Gallery Problem

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Grade level: secondary (Years 9-12)
Subject: mathematics
Time required: 45 minutes

Activity Overview

The question of where patrons should stand to enjoy the best view of a painting is one art gallery curators must consider on a regular basis. In this particular case we will consider a painting that is two meters tall that has been placed 1 meter above the average person's eye level. Where should the average person stand in order to get the best view?

Concepts

- *Algebraic and Geometric modeling*
- *Data representation and interpretation*
- *Angle measurement and inverse trigonometric functions*
- *Optimization using numeric, functional and differential calculus based approaches*

Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to measurement of distances and angles, and inverse trigonometric functions. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build a new and deeper understanding.

- *At the Algebra 2/Precalculus level, this activity can serve to consolidate earlier work on trigonometric functions. It offers a suitable introduction to exploring trigonometric data, model fitting using inverse trigonometric functions, and interpretation of graphs.*
- *The screenshots on pages 2–4 demonstrate expected student results.* *Refer to the screenshots on page 5 for a preview of the student .tns file.*

Classroom Management

- *This activity is intended to be a* ***cooperative problem solving experience*** *with students in* ***small groups****. You should seat your students in pairs so they can work cooperatively on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Students will engage the task using their handhelds, although the majority of the ideas and concepts are only presented in* ***this*** *document; be sure to cover all the material necessary for students' total comprehension.*
- *The student worksheet is intended as an investigation through the main ideas of the activity. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on a separate sheet of paper, or just use the questions posed to engage a class discussion.*
- *Suggestions for optional extension questions are provided at the end of this activity.*

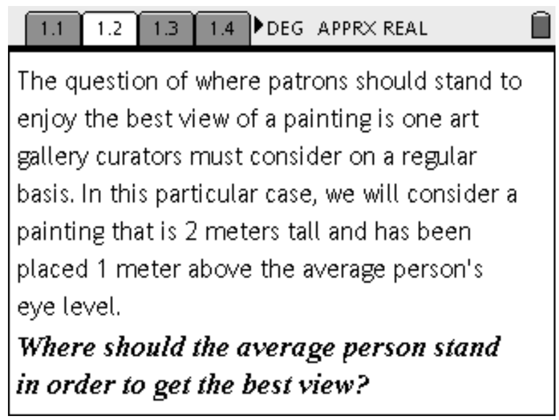
TI-Nspire™ Applications

Calculator, Graphs & Geometry (G&G), Lists & Spreadsheet (L&S), Notes

The Art Gallery Problem

The problem of determining the best position to view a painting hanging on an art gallery wall has been contemplated since at least the time of Regiomontanus in 1471 (Maor, 1998). The essence of the problem lies in finding the distance, x, a person should stand away from the wall that will optimize the viewing angle of the art work. Use of the multi-representational facilities of TI-Nspire has the potential to provide students with the opportunity to engage in an authentic mathematical modeling activity. The mathematics associated with non-contrived applications becomes complex very quickly. The facilities of TI-Nspire permits the management of this complexity and so allows students to engage with life-like problems and the mathematization of these problems via the process of modeling.

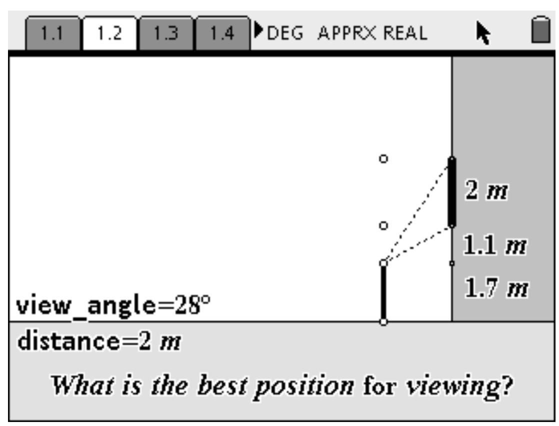
Step 1: Students should role-play the problem by choosing a suitable piece of art work around their classroom or somewhere else in the school and try to find the best place to stand by simple trial and error. This experience will allow students to appreciate the fundamental ideas behind the problem they are trying to solve.



After the initial experience described above students should work in groups to assist each other in fully understanding and then mathematizing the problem. Make it clear to students that you expect them to solve the problem in more than one way but that you also expect all members of the group to contribute the development of each solution.

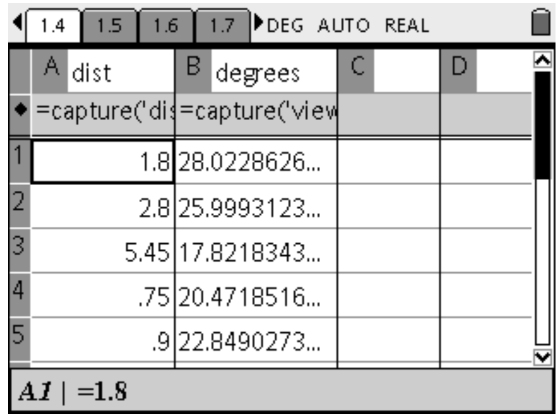
Step 2: After modeling some “real-world” exemplars of the problem, students are ready to work with the dynamic geometric model provided.

By varying features of this model, students should attempt to describe their initial observations and make conjectures based on their intuitive feel for the situation – they should observe that the viewing angle varies with the distance of the viewer from the wall. In fact, they may notice that it appears to increase to a maximum value at around a distance of 2 meters from the wall.

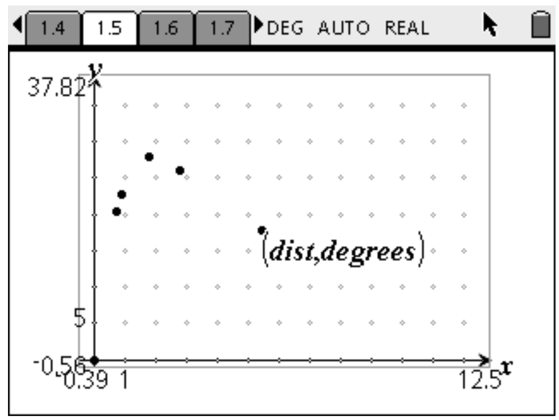


Step 3: By pressing ctrl + . at different distances from the wall, students may manually collect data from their geometric model. This may be inspected on the *Lists and Spreadsheet* page provided (page 1.4) and then viewed graphically on the next page (1.5).

Students will need to collect data at sufficient points to build a suitably detailed graph from which to make conjectures and to draw conclusions.

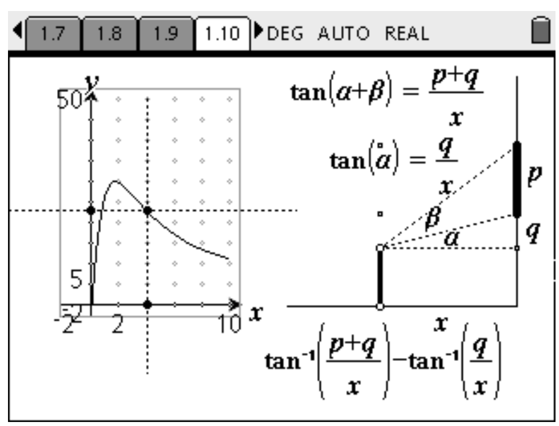


Data collected in this way will provide enough detail for students to estimate an optimal distance from the wall, but should point them to the need for an algebraic approach if a more accurate solution is desired.



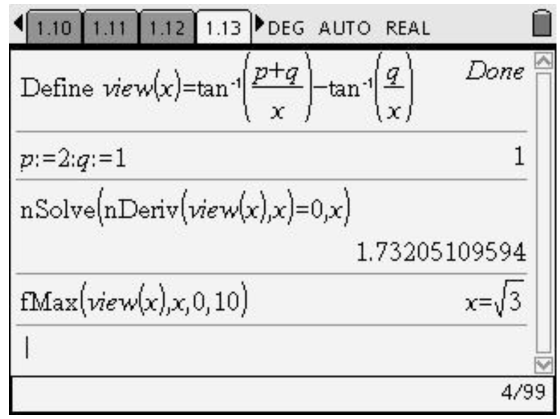
Students may use the geometric model, the table and the graph to answer specific questions and demonstrate their understanding of the model, including an estimate of the best viewing position.

Step 4: They may then use the geometric model to build their algebraic function using the Text Tool to label the diagram and to develop the algebraic parts of the final function. This may then be verified graphically as shown.



Step 5: Finally, using either formal or informal calculus techniques, students should attempt to compute the optimal position for viewing a work of art as described.

For an informal approach, the numerical function maximum may be applied to the defined function; for a formal calculus approach, solving to find the value at which the derivative equals zero would be most appropriate.



(*TI-Nspire CAS screen shown)*

Assessment and evaluation

Students should be responsible for the preparation of a report that provides details of how their group solved the problem they investigated. The report should also include, however, details of at least one other approach that was developed by students from another group. This will encourage students to learn from their peers and to be actively engaged in the process of supportive critic during the presentation of other groups' approaches and solutions.

Scaffolding questions for the Art Gallery problem

<table>
<tr><td>1)What is the angle between the patron's eye and the top and base of the painting when the patron is standing 3m away?</td><td>1) 0.464 radians (26.6 degrees)</td></tr>
<tr><td>2)What is the angle between the patron's eye and the top and base of the painting when the patron is standing 1m away?</td><td>2) 0.464 radians (26.6 degrees)</td></tr>
<tr><td>3)Fill in the table below?
<table><tr><td>Distance (m)</td><td>0.5</td><td>1.0</td><td>1.5</td><td>2.0</td><td>2.5</td></tr><tr><td>Angle (rad)</td><td></td><td></td><td></td><td></td><td></td></tr></table></td><td>3)
<table><tr><td>Distance (m)</td><td>0.5</td><td>1.0</td><td>1.5</td><td>2.0</td><td>2.5</td></tr><tr><td>Angle (rad)</td><td>0.32</td><td>0.46</td><td>0.52</td><td>0.53</td><td>0.5</td></tr></table></td></tr>
<tr><td>4)Use this table to estimate the best position for the patron to stand in front of the picture.</td><td>4) Between 1.5 and 2.0 metres.</td></tr>
<tr><td>5)Develop a formula for your model that describes the relationship between the distance away from the painting and the angle that exists between the patron's eye and the top and bottom of the painting.</td><td>5) $view(x) = \tan^{-1}(\frac{3}{x}) - \tan^{-1}(\frac{1}{x})$</td></tr>
<tr><td>6)Use your function and graph to determine the best position to stand away from the painting.</td><td>6) Approximately 1.7 metres.</td></tr>
<tr><td>7)Confirm this result by making use of the calculus facility of Nspire.</td><td>7) Distance = $\sqrt{3}$ metres.</td></tr>
</table>

Extension

The growth in popularity of flat-screen television screens which may be hung on a wall lends new relevance to this problem, since students are likely to be very interested in finding the optimal height and position from which to enjoy their widescreen viewing. In this case, measurements would be made from the preferred viewing position (lounge or armchair) and use this to calculate the best height for the screen.

