TI-nspire™

The impact of *b* in $f(x) = ax^2 + bx + c$

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Grade level: secondary (Years 9-12)
Subject: mathematics
Time required: 90 minutes

Activity overview

In this activity you will examine the influence parameter *b* has on the quadratic function. You will use different techniques to visualise this. You will use CAS to find the vertex coordinates. You will plot the vertex points of *f*(*x*) for different values of *b* and look for a pattern. You will use regression to determine a function which fits this pattern. You will use sliders so that you can see dynamically that your conjecture is true. You will use CAS to test your guess.

Background

Students are generally subjected to the general quadratic equation of the form $f(x)=ax^2+bx+c$ *before being introduced to the transformational form* $f(x)=a(x-h)^2+k$*. It is in the earlier format that the axis of symmetry is initially learned using a given rule rather than being investigated as this task allows.*

Concepts

Algebra, graph drawing, statistical plotting and regression.

Teacher preparation

It will be useful for teachers to have looked at the effect of parameters a and c so that students also feel the need to know what parameter b represents. It may be useful for students to work in small groups but each having a different starting functions. This will allow them to support one another and also allow them to collectively form a possible conjecture for the general case based on their individual findings. This then saves the individual student having to complete Stage 6 of this task.

Technical prerequisites

Students should know how to:

- Use the point on, the point of intersection, line segment, perpendicular, midpoint, locus, transfer measurement, text, length and calculation tools.
- Use the define function.
- Use formulas in the spreadsheet and plot data.

Step-by-step directions

Stage 1:

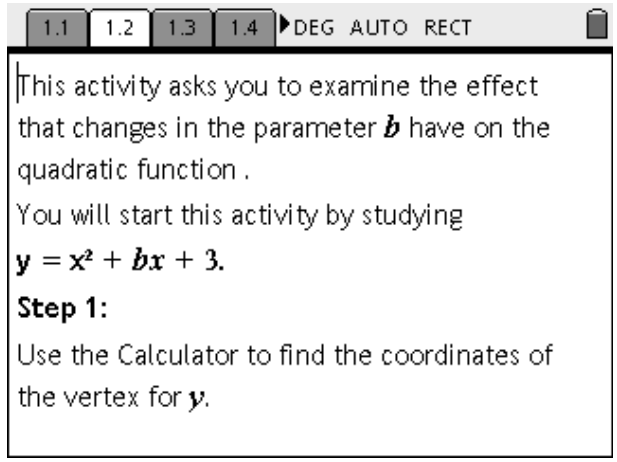
Give students the task of finding the coordinates for the vertex of $f(x) = x^2 + bx + 3$ using CAS (if available).

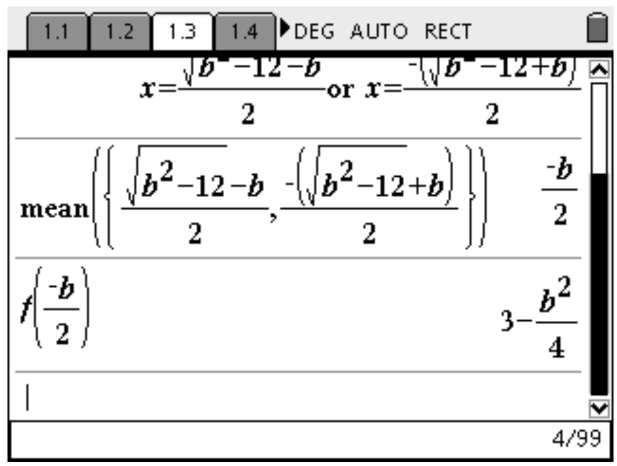
Since *a* is to be held constant, the shape of the parabola remains unchanged. To see what happens if *b* is altered, study how the vertices move for different values of *b*. Then plot these points on a system of coordinates.

1. Open the **Calculator** application.

2. Define $f(x) = x^2 + bx + 3$ (Type $b \times x$).

3. Either determine the axis of symmetry by finding the midpoint of the roots (solving *f*(*x*)=0), or use $x = \dfrac{-b}{2a}$, hence calculate $f(\dfrac{-b}{2})$.
   (Refer to screen opposite for the CAS solution)





(TI-Nspire CAS screen shown)

Stage 2:

Use the spreadsheet to calculate the coordinates for different values of *b*.

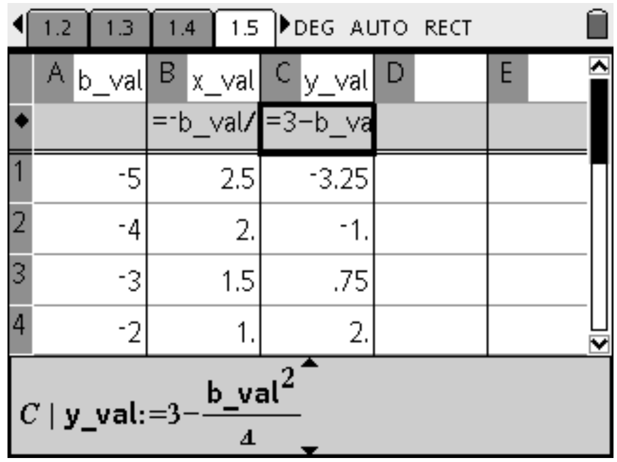
1. Open the **Lists & Spreadsheet** application.

2. Enter the values for *b* from -5 to 5 in column A. (or use **seq** command: **seq(X,X,-5,5)**)

3. Place the cursor at the top of column B and press enter. Type *b_val* and press enter.

4. On the next line, press [Enter] and type =-b_val/2
   You have now calculated the *x*-coordinates for the vertices (label the column x_val).

5. Repeat the process for a column labelled *y_val*. The formula you use now is =3 – b_val^2/4. You now have the y-coordinates for the vertices.

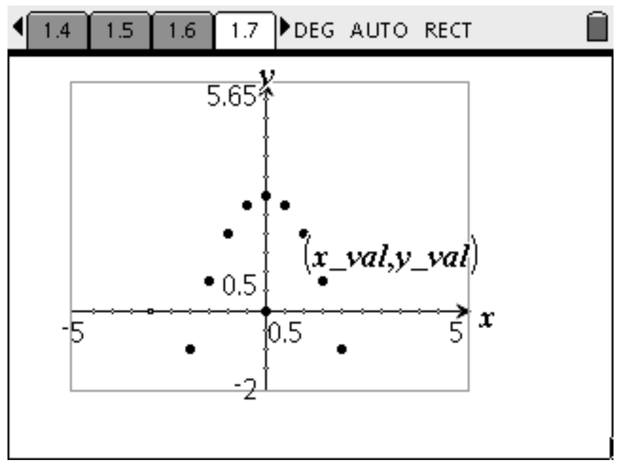


Stage 3:

The students should plot these points.

1. Open the **Graphs & Geometry** application.

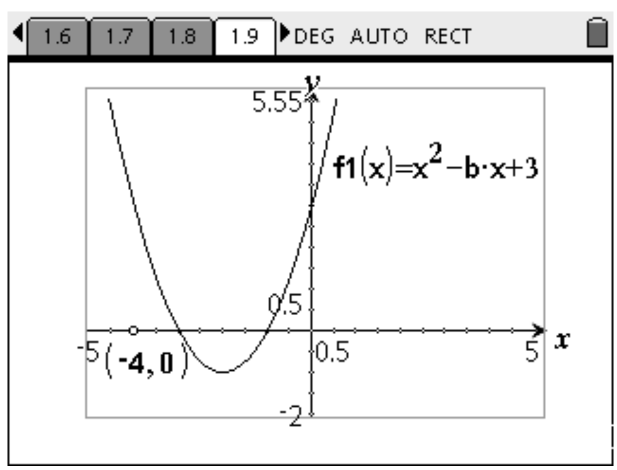
2. Choose **MENU > Graph Type > Scatter Plot** and then use the drop-down menus for *x*- and *y*-lists. Click on the *x*-list and select *x_val* in the dialogue box which appears. Go to the *y*-list and select *y_val*. Press [Enter] and you will have plotted the **vertices** of the parabolas generated as *b* changed from -5 to 5, steps of 1.



Stage 4:

It is now time to draw in the function $f(x) = x^2 + bx + 3$ for different values of *b*.

1. When you grab and move the variable (slider) point on the axis the *"length value"* will alter. This will be used to change the *b*-value in the following steps.
2. Now move the slider and see what happens.
3. Select [Text] and type in $x = \frac{-b}{2}$ and $y = 3 - \frac{b^2}{4}$. Select [Calculate] and work this out using the *coordinate value* for b.



NOTE: To increase and decrease the numerical display accuracy, hover the mouse over the value and press + or -.

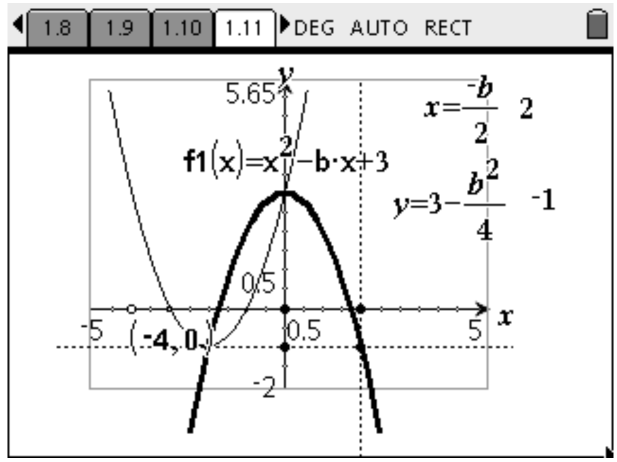
4. Select **MENU > Constructions > Measurement Transfer** and click on the *x*-value then click the *x*-axis. Repeat using the *y*-value on the *y*-axis.

5. Construct the perpendiculars through the *x*-value and through the *y*-value.

6. Select **MENU > Points & Lines > Intersection point** and point to the lines, one at a time, that you have now drawn. Then mark the point of intersection between these two lines.

7. Select **MENU > Construction > Locus** and click on this point of intersection, and then on the slider point on the x-axis.

8. By plotting this, you have now obtained the required parabola.



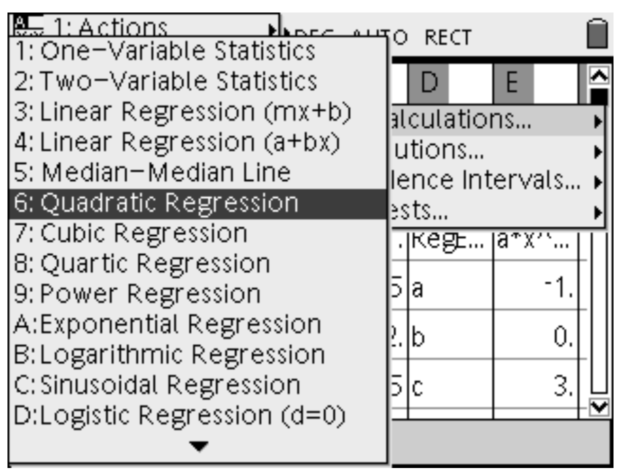
Example shows:
f(*x*) with *b* = -4, vertex (-2,-1)
NOTE the “m” which indicates a “minimum” point.

Compare this parabola with that of the statistics plot of the vertices done earlier.

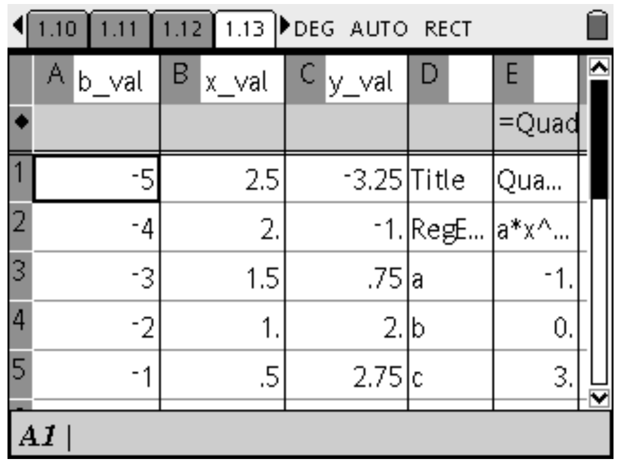
Stage 5:

You will now find an expression for the function which best matches your plot.

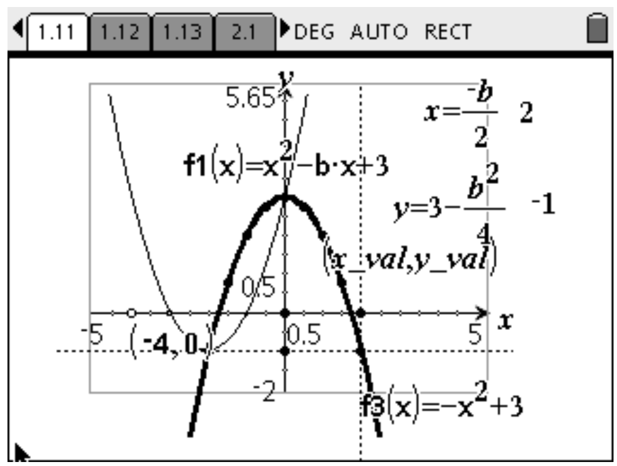
1. Go to the **Lists & Spreadsheet** application.
2. Select [Statistics] and select Quadratic regression.
3. Type *xb* in the *x*-list and *yb* in the *y*-list. Press OK.



4. You will now find the values of *a*, *b* & *c* for the parabola (see screenshot).



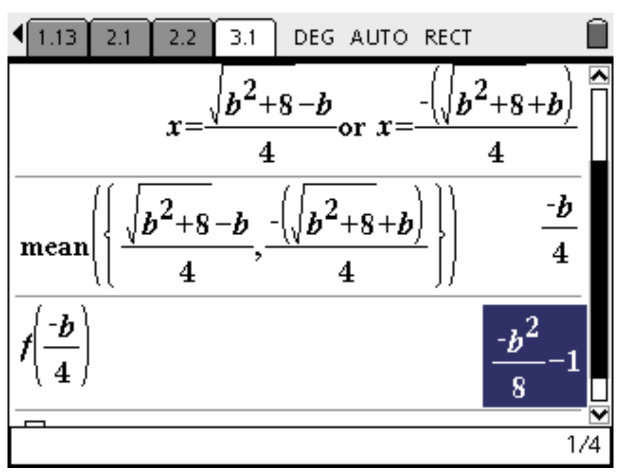
5. Now go to the **Graphs & Geometry** window once again. Enter the function $y = -x^2 + 3$ which you have just found.(You can type the function in a text box, press Enter and drag it onto the axes or enter in the Graph box at the bottom of the application.



This gives us the basis for guessing that the general function we are looking for has the rule $y = -ax^2 + c$ *.*

Stage 6. *If students can share results from different starting functions then this section can be omitted.*

1. Repeat the above procedures using other similar functionns, such as $f(x) = 2x^2 + b \cdot x - 1$
2. You should add a new **Problem Space** and so avoid using previously defined variables.



Do you now have the basis for guessing that the general function we are looking for has the rule $y = -ax^2 + c$ *? * Allow students to form this postulate for themselves.*

Stage 7: General case.

You will now show that the vertices for the parabolas you get when you allow *b* to pass through all possible values, will lie on the function $y = -ax^2 + c$.

1. Insert a new Problem and open the **Calculator** application.

2. Define $h(x) = ax^2 + bx + c$.

3. Find the coordinates of the vertex of this function as in Stage1. (refer to screen opposite)
   Let function *g* be the path of the vertices of *h(x)*.
   Define this function in terms of *b*.

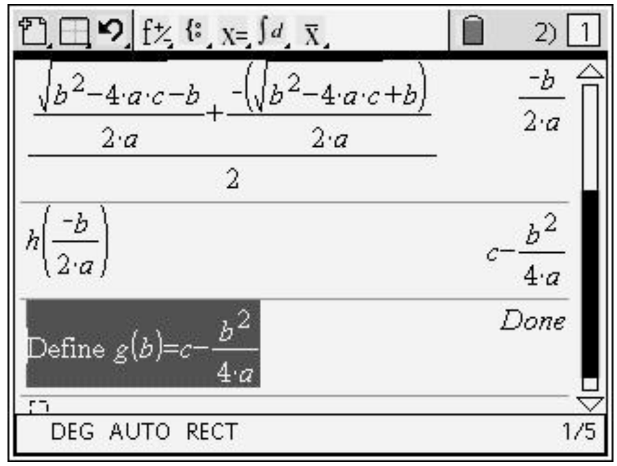
4. Define $g(b) = c - \dfrac{b^2}{4a}$

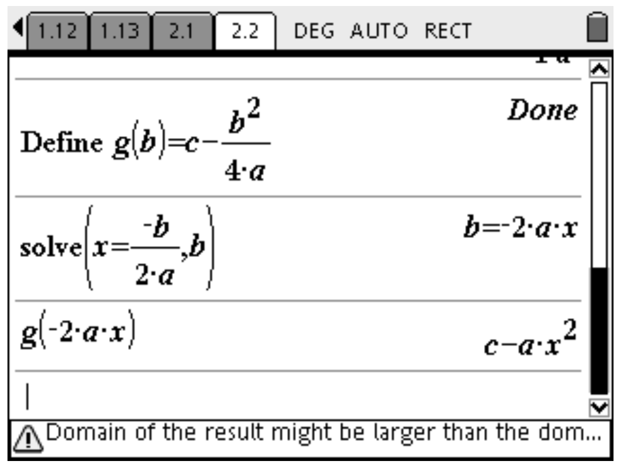
5. To get *g* in terms of *x* let $x = \dfrac{-b}{2a}$
   i.e. the *x*-coordinate of the vertices.

6. Solve the above equation in relation to *b*.

7. Calculate $g(-2ax)$.

Is the postulate supported by the general solution?





(TI-Nspire CAS screens shown)

Assessment and evaluation

- Students should be encouraged to write a report explaining the task and the reasoning behind their discoveries. Student reports of this investigation should be clear and detailed; observations should be documented with appropriate screen dumps and clear explanations of each representation (tabular, geometrical, graphical and algebraic) should be required. Questions posed throughout the task should be addressed with appropriate reasoning.

Activity extensions

- Use the sliders for parameters *a* and *c* as well, and note that you can select all three parameters independently of one another.

- The same technique can be used to examine other functions, for example $y = y_0 + a\sin(b(x - v))$.