The Chicken Run

by Vince Geiger

Activity overview

A farmer has 60 metres of chicken wire and wants to build the largest rectangular run he can. He has two options. The first option is to build a free standing run in which he will use the fencing at his disposal to enclose the four sides of the run. The second option is to build the run up against an existing fence and use the available fencing to enclose the area with three sides additional to the existing fence. The farmer wishes to know which dimensions will offer the greatest floor space for each model as well as which design will provide the greatest area for the chickens to walk around in.

Background

The question of optimizing the area enclosed by an invariant perimeter is not new but the ideas that the enclosed area will vary with the dimensions of the enclosure is almost anti-intuitive to many students (and some adults).TI-Nspire permits a much more integrated, multi-dimensional view of this problem that is often only investigated using the device of calculus.

Concepts

Scale, Geometric construction, Co-ordinate Geometry, Algebraic Modeling, Function Graphing, Optimisation using numeric, functional and differential calculus based approaches

Teacher preparation

Allowing the students the opportunity to "experience" the problem scenario is very helpful in it orientates students thinking toward the nature and the goals of the task. Thus some work outside with tape measures and wooden stakes or tent pegs would be a useful start to this activity.

Classroom management tips

There is a great deal of value in making use of a group or cluster approach to solving this problem because of the discussion and debate that can be stimulated when groups have to present, argue for, and sometimes defend their findings. Students should work in clusters with half the clusters working on the fully enclosed run and the other half working on the run that is to be built up against a wall.

Students should spend some initial time working in their groups to develop an understanding of the problem and also to develop a strategy for how to proceed. If they reach a point when they are genuinely stuck they should send one of their members to seek advice from another group and bring this back for discussion. The teacher should have minimal input other than general hints and helping out with any technical problems students encounter when using technology.

Towards the end of the available time each group should present their findings to the class using the TI-Nspire device and projection facilities as a way of supporting their arguments. The responsibility of the class during this phase is to question and challenge (in a supportive way) the solution offered by each group and to assist in the correction of errors and misconceptions.

Finally, the teacher should summarise all the ideas that have been developed during the session, making sure to identify any gaps and leading students to filling these in. Students should then complete a report that summarises the finding of the class as well as the different approaches that may have been used to develop those findings.

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Grade level: Late primary/secondary (Years 6 – 11)
Subject: mathematics
Time required: 90 minutes

Technical prerequisites

Students should know how to:

- *Construct basic geometric objects (points, lines, segments, circles…) and use features such as Measurement Transfer, Locus and Calculate. (**Graphs & Geometry** Tool)*
- *Transfer Measurements to co-ordinate axes so as to link x- and y-co-ordinates from a geometrical construction to a function graphing environment. (**Graphs & Geometry** Tool)*
  (NOTE: the geometric constructions are NOT required for this task unless the teacher specifically wishes students to create their own constructions. Generally, students would use the geometric objects provided with which to explore the problem).
- *Store values from geometric constructions as variables, which can then be used in algebraic constructions. (**Calculator**)*
- *Define and graph functions which can then be used to graphically verify an algebraic model against a geometric locus. (**Calculator** and **Graphs & Geometry** Tool)*
- *Define and operate upon expressions as preparation for solving relevant equations using CAS (**Calculator** Tool)*

Step-by-step directions

Stage 1: Understanding the task and preliminary exploration.

1) Describe the task and ask students to act out the problem while attempting to develop an approximate solution with the assistance of physical aids such as tape measures, tent pegs, etc.

2) Provide two specific focuses for their investigation.
   i) What happens to the area of an enclosed chicken run as you change the dimensions of the rectangular enclosure?
   ii) What dimensions will provide the maximum area for the chickens to walk around in for each of the suggested models?

3) Explain how the geometric objects provided within **Graphs & Geometry** work and how they can be used to model the two scenarios they have been asked to investigate.

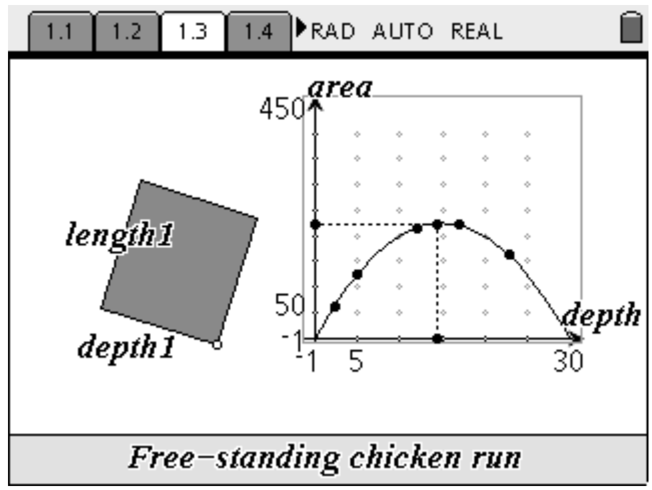
4) Provide opportunity for preliminary explanation and the chance to ask for clarification.

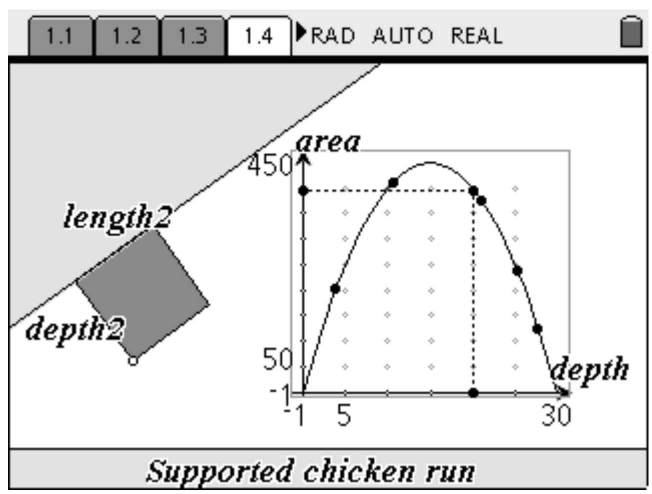
Stage 2: Geometric investigation: Exploring a geometric model in order to develop a numeric model and then building a graphical representation from the numeric model.

1) Students should explore the focus questions using the geometric models provided. For older or more experienced students a discussion of the nature of the constructions behind these models might be of value. This is especially the case if there is an intention to extend students by challenging them to develop other models, i.e., other shapes or orientations.

2) The teacher should not provide a great deal of input at this stage although if students are struggling they might be directed to the idea of pressing CTRL-(.) (period) in order to build a more detailed graphical model. Students could be encouraged to refer to the organized list of these values in the **Lists & Spreadsheet** application and to infer a solution from these values.

3) After looking carefully at the numeric values they have generated, students could make use of the *Locus* tool to identify the curve of continuous values that represent this problem. This curve can then be used to identify the dimensions that will optimize the floor space of either model. Further, this stage of the process should also suggest to students the possible algebraic models which might be appropriate for each respective circumstance.





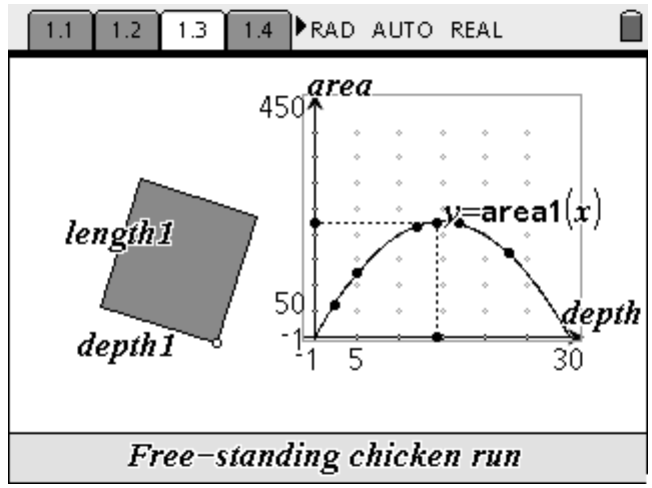
Stage 3: Taking a functional approach to the task

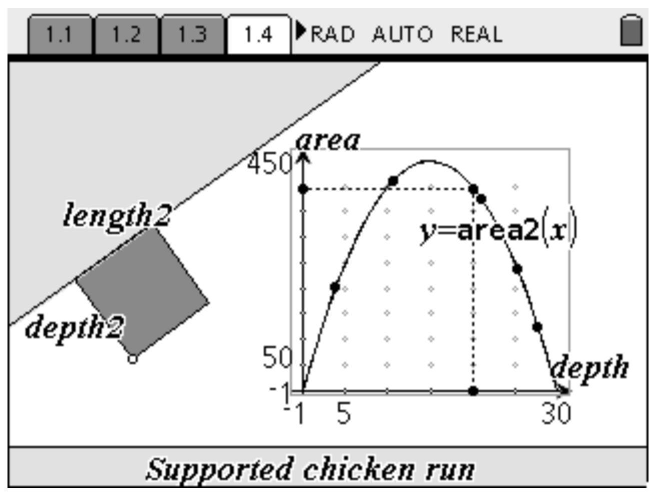
1) Students can make use of the variable linking facility to develop and test algebraic models against the geometric models they develop initially. These can be tested by altering the equation "live" until the model generated by the geometric approach matches that of the algebraic conjecture.

2) Alternatively, students can develop their algebraic model from first principles and enter their equations directly into the function entry line to produce a graph of their model(s). These models can be verified to see if coordinates on their functional model match those of their geometric model.

3) The *Point On* tool can then be used to identify the local maximum and so the optimum dimensions of their chicken run.

4) Students should be constantly making comparisons between the approaches they have used so far in order to confirm their findings as they will soon have to offer these to their classmates in a public fashion.



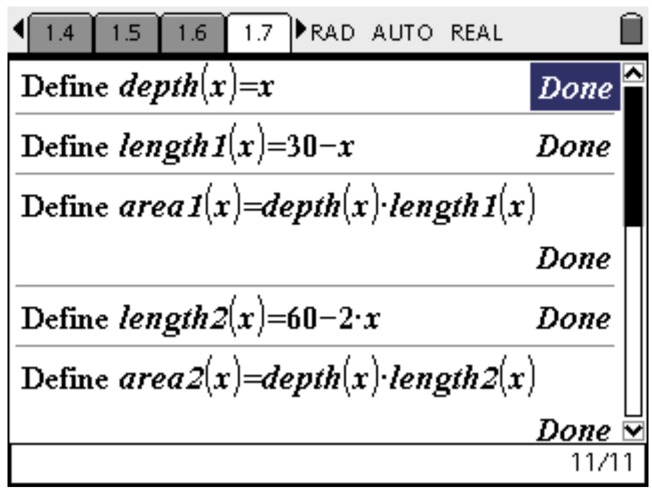


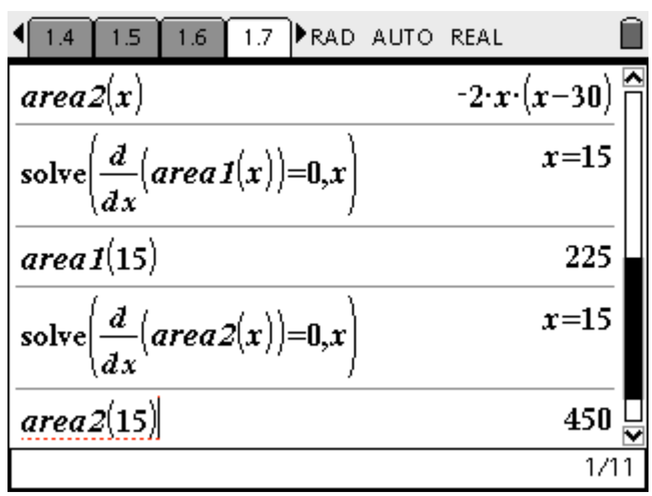
Stage 4 Confirming a result through the use of CAS

1) Finally, students can confirm their findings (again) through the use of Nspire's CAS facility (if available). Linking variables from **Graphs & Geometry** makes it easier to enter expressions.

2) At this stage students should present the finding of each group making use of TI-Nspire as a presentation tool to support their conjectures.

3) This presentation should be followed by a report that discusses their findings and also uses different representations of the problem to confirm their solution. The report should also discuss the strengths and limitations of their models as well as any assumptions that have been made in the construction of the models.





(TI-Nspire CAS screen shown)

*Note 1: In order to make use of the data linking facility across applications a consistent scale must be used across all applications. It is likely students will need to be reminded of the need to take account of scale.
*Note 2: This activity can be used at many school levels. In the later years of primary school/early secondary students could use of pre-constructed geometric models that have already been developed in investigate the problem numerically. As students progress through school other elements can be introduced such as a graphical approach or a CAS approach. It will be left to the teacher to make judgements as to when the different elements can be introduced.

Assessment and evaluation

This task would be best assessed through a report or assignment. Students will need to explore and investigate the task and then make use of a variety of TI-Nspire's facilities to confirm their findings and this is difficult to do under supervised exam conditions. As this is a modeling activity students should be directed to consider and discuss issues such as the assumptions made in setting up their models and the strengths and limitations of those models. The report should address both the problem they have investigated personally as well as the alternative task presented to them by other groups.

The presentation component of the task could also be used as a basis for what students have learnt. In this assessment mode care should be taken to ensure that each member of the group has as meaning ful task within the presentation where they can demonstrate their knowledge of content as well as their understanding of the approach they tool to solving the problem. Each student should also have a role in the presentation that permitted them to demonstrate their facility with TI-Nspire.

Scaffolding questions for the Chicken Run task

For your model:

1) What is the length of the chicken run when the width is 10 meters? When the width is 15 meters?

Answer: 1) Stand alone model: 20 m, 15 m
Three sided model: 40 m, 30 m

2) What is the area of your chicken run when the width is 10 metres? When the width is 15 metres?

Answer: 2) Stand alone model: 200 m^2, 225 m^2
Three sided model: 400 m^2, 450 m^2

3) Fill in the table below?

Width (w)	5	10	15	20	25	30
Length (l)						
Area (a)						

Answer: 3) Stand alone model

Width (m)	5	10	15	20	25	30
Length (m)	25	20	15	10	5	0
Area (m^2)	125	200	225	200	125	0

Three sided model

Width (m)	5	10	15	20	25	30
Length (m)	50	40	30	20	10	0
Area (m^2)	250	400	450	400	250	0

4) Use these tables to estimate the maximum area of the floor plan of your chicken run.

Answer: 4) Stand alone model: 225 m^2
Three sided model: 450 m^2

5) What is unusual about the last entry in the table? Can you explain why this is?

Answer: 5) The area is 0. Because the width makes use of all the available fencing the length is 0. Since A=l⋅w then the are is 0 as well.

6) Develop a formula for your model that describes the relationship between width and area.

Answer: 6) Stand alone model: $a = w\frac{(60-2w)}{2}$
Three sided model: $a = w\frac{(60-w)}{2}$

7) Use your equations and the equation grapher to determine the dimensions that will provide your chickens with the maximum amount of floor space?

Answer: 7) Stand alone model: 25 m x 25 m: 225 m^2
Three sided model: 15 m x 25 m: 450 m^2

Activity extensions

1) Find general rules for the models studied above (one for each) that will predict the dimensions needed to optimize the floor space given the length of fencing available.
2) Find another rectangular orientation that could be used to build the chicken run and investigate it in the same ways as above. (hint: what other existing structure could be used to help support the available fencing.
3) Explore a range of other shapes as the basis of the floor plan of the chicken run. Determine which of your chosen shapes offers the best area to perimeter ratio. Can you make a general statement about the shape of a floor plan and the floor plan's area to perimeter ratio.