TI-*nspire*™

# Building Concrete Foundations to Algebra

by: Stephen Arnold
Grade level: Junior secondary (Years 7-9)
Subject: mathematics
Time required: 45 to 90 minutes

## Activity overview

*The materials for this activity offer a carefully sequenced introduction to the symbolic notation of early algebra, using concrete manipulatives (“algebra tiles”).*

## Background

*Concrete approaches to building student understanding and manipulative skills in early algebra have been well established through research and successful practice. The approach offered in this activity should be used subsequent to pattern building activities and jointly with concrete manipulatives (cardboard shapes are quite suitable).*

## Concepts

*Algebraic simplification, factoring and expanding.*

## Teacher preparation

*Students should begin their introduction to algebra through pattern building using number. Tables of values are most suitable, supporting students in learning the skills needed to move from a vertical approach (“add the next three terms to the pattern: 2, 5, 8, …”) to a more “horizontal” one: Find the pattern which changes* **x** into **y**:

| | | | | | | |
|---|---|---|---|---|---|---|
| X | 0 | 1 | 2 | 3 | 4 | 10 |
| Y | 3 | 5 | 7 | 9 | 11 | 23 |

The concrete model used in this activity should be accompanied by students’ own cardboard equivalents for the unit square (“one”), the “x” and the “x squared” in two colours: one positive and one negative.

## Classroom management tips

*After a careful introduction of the materials by the teacher, students may be encouraged to work in pairs and to build models for each other, learning to move easily between concrete and symbolic representations. The additional use of both numerical and graphical representations (using the combined Lists and Spreadsheet and Graphs and Geometry page) should accompany the concrete and symbolic forms available.*

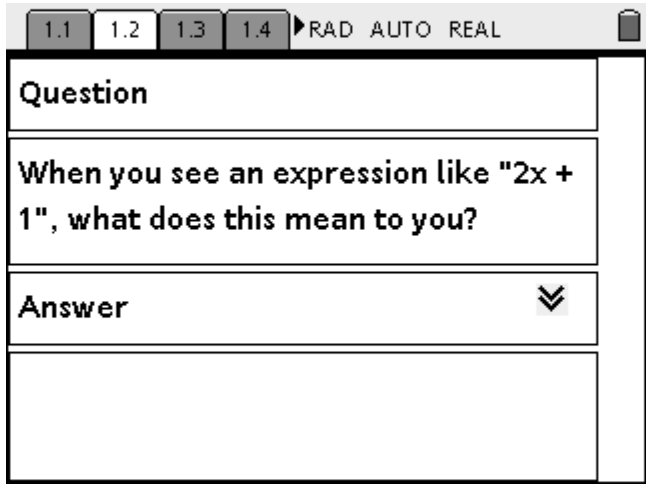
*Further, teachers need to ensure that the foundational understanding of* ***variable*** *is clearly established at this point: students should frequently re-define the values of* ***x*** *and attempt to predict values for the various expressions using these new values for* ***x****. The static approach involving cardboard shapes is deficient in drawing no explicit link with other representations and in the value of the variable being fixed once they are made. Both these failings are well countered in this interactive approach and student understanding of the key concept of variable developed strongly and consistently.*

## Technical prerequisites

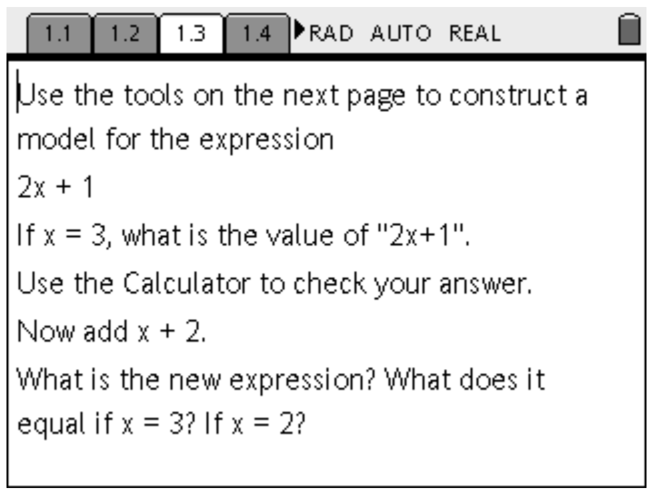
Students should know how to:
- *Enter algebraic expressions using the Calculator, and enter formulae using Lists and Spreadsheets.*

## Step-by-step directions

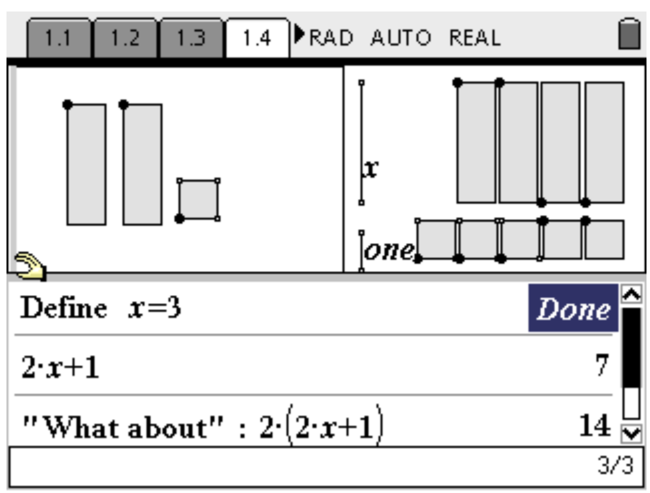
While this activity is primarily intended for those first beginning symbolic algebra, there may well be value as a consolidation and concept development activity for older students who may not have been exposed to these "concrete approaches". Ideally, students should have a rich and varied representational image for algebraic expressions, such as "2x + 1". This should include "function machine" imagery (a value goes in, another value comes out), tabular and graphical thinking, an active process (2 lots of **x** plus 1) as well as some form of symbolic thinking which is meaningful in some way. The multiple representations offered here are intended to assist in the development of such rich thinking about algebraic objects, expressions and, later, equations.




It is important at all times to stress that, while the symbols of algebra (such as **"x"**) may appear to be objects to be manipulated according to certain rules, they are, in essence, placeholders for a potentially infinite array of numerical values. The emphasis of this activity is to simultaneously build clear understanding of the rules of addition and subtraction of algebraic expressions, while realizing that every algebraic expression has a value which varies as the value of **x** (or whatever the variable happens to be).

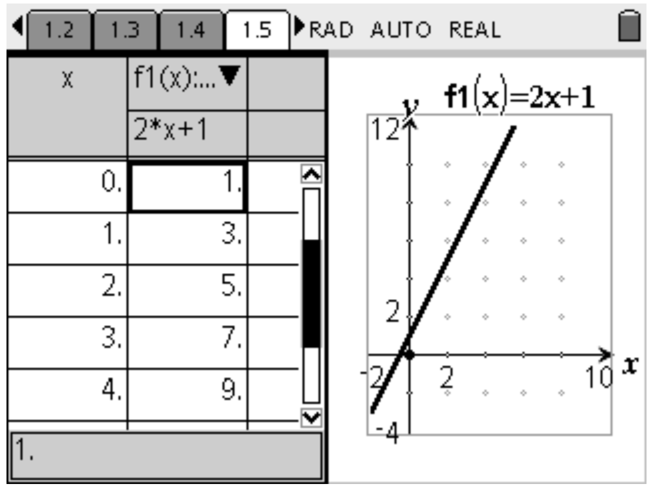



Simple algebraic expressions are offered for students to construct using the "**x**" blocks and the unit squares and, having done so, to enter differing values of **x** and attempting to evaluate the expression for each. This may then be checked by entering the expression symbolically in the accompanying Calculator tool, and evaluating it for different **x** values, as defined.



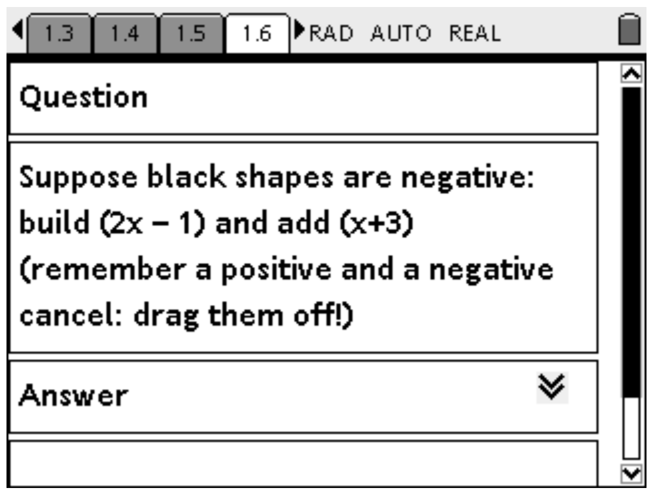

 

Students should be encouraged, for each expression they build concretely, to use the Calculator for symbolic evaluation, Lists and Spreadsheet for numerical (tabular) representation, and the accompanying scatter plot for the graphical representation. Comparison with peers and small group and class discussion should be a part of this process.

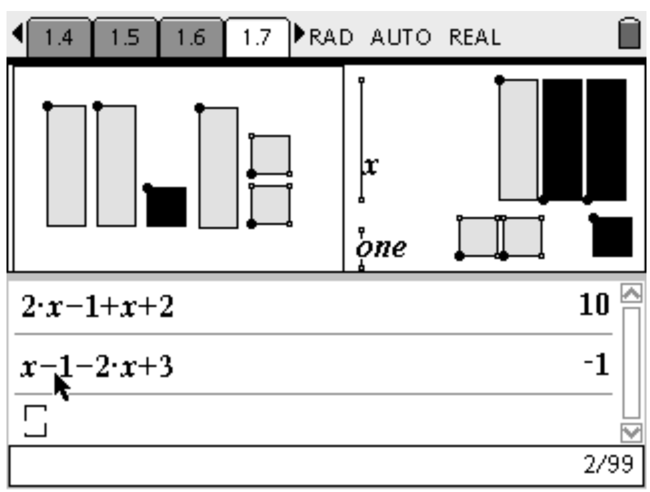



Once the teacher is satisfied with positive valued models, the idea of negative values may be introduced. This simply involves students removing from the workspace any equivalent positive and negative shapes, which "cancel" each other out.

In the example shown **(2x – 1) + (x + 2)** students should observe that the negative unit square should be dragged off the workspace along with one positive unit square, leaving the result **3x + 1**.

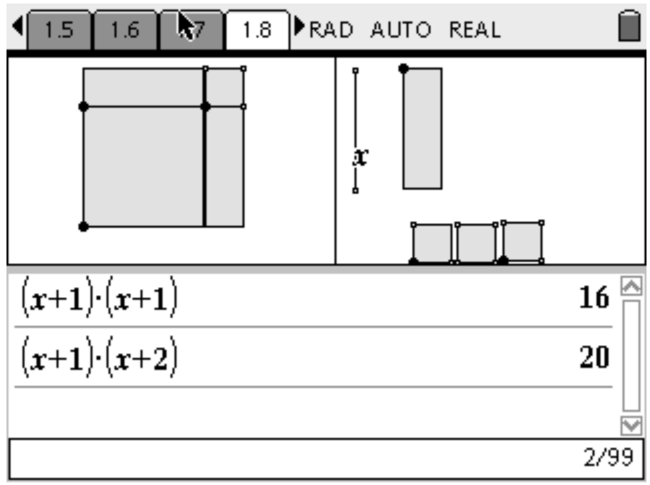



This equivalence should be tested symbolically using the calculator: students should enter both the expanded and simplified forms and observe that these remain numerically equivalent for various values of **x**. This equivalence may also be verified using table of values and graphical models.



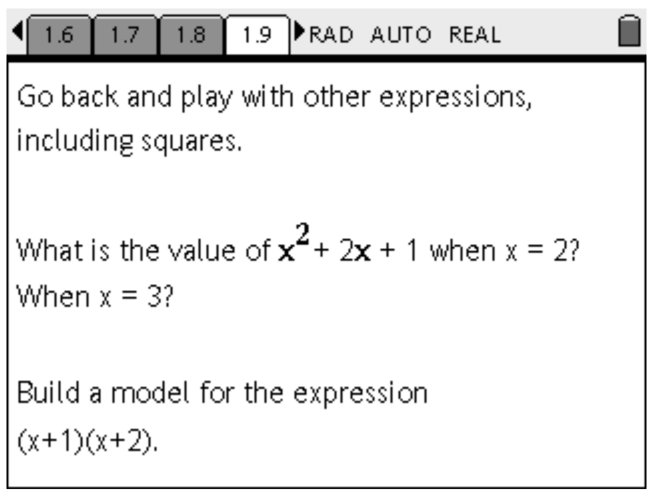

Students should also be exposed to the factored form for simple expressions involving a common factor: thus,**2 lots of x + 2** becomes **2(x + 2)** and is easily seen to be equivalent to **2x + 4**.

NOTE the significance of the use of *language* here. The form “lots of” to signifiy multiplication should be used repeatedly: this **2x + 1** is always **2 lots of x plus 1**; the factored form **3(2x + 1)** becomes **3 lots of (2x + 1)**. Such carefully chosen use of correct mathematical language plays a key role in building strong foundations and clear understanding of the processes of algebra (and of numerical operations!).



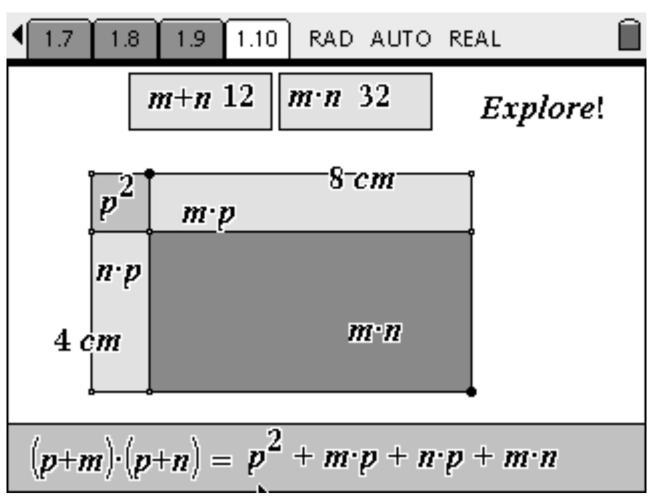

The model may finally be extended to include the **x squared** shape, identified as the area of an **x by x square** and represented symbolically as $\mathbf{x^2}$.

This is further extended to support binomial products, including **(x + 1)(x + 2),** readily seen to be equivalent to $\mathbf{x^2 + 3x + 2}$, as shown.




Finally, the general binomial product representation is offered, and students should be supported in exploring the relationships between the product values and their expanded form. Potentially, this model extends the range of values from the usual whole number examples encountered in text books to the range of decimal forms also.

While the number of concrete models available throughout this activity is limited by the screen size to relatively small numbers, the *TI-Nspire* based activity is intended to provide only a starting point from which students may readily extend using, first their own concrete models (cardboard cut-outs) and, when ready, simply the symbolic forms which will then have meaningful and firm foundations.



## Assessment and evaluation

- *Once students have worked sufficiently through the interactive models available here, these understandings and skills should be readily transferable to any of the usual textbook and class work examples available for early algebra. Initially, teachers should encourage students to accompany symbolic (algebraic notation) forms with their own sketches of equivalent concrete models. Eventually, this practise may be discarded since students will have solid cognitive representations to support their symbolic work.*

## Activity extensions

- *The concrete models used here may be readily extended to linear equation solving, using not one but TWO workspaces. A very effective approach involves "cups and blobs" and two desks at the front of the room. Two rules are given: the number of blobs in each cup is always equal - call this value "**x**", and the total number of blobs on each desk is always the same. Three cups (containing an unknown but equal number of blobs) and two extra blobs may be placed on one desk; one cup and six blobs are placed on the other. The symbolic form may be written on the board:* **3x + 2 = x + 6**. Since removing the same thing from each desk keeps the total number of blobs equivalent, students may then see that removing two of the "free" blobs from each desk keeps everything equal (**3x = x + 4**). Then removing one cup from each desk is seen to leave **2x = 4**. Separating the contents into two equal parts shows readily that **x = 2**. Once done several times in this manner, the same algebra tile approach as for the current activity may serve to build strong understanding of the equivalences involved in the linear equation solving process.