Ian Edwards
Grade level: secondary (Years 11-12)
Subject: mathematics
Time required: 45 minutes

BLACK BEAR CUBS – Birth to One Year Old

Background:

Black bears are found in all but a few eastern counties of Virginia, but the densest populations are in the western mountainous part of the state and in the south east corner, mainly the Great Dismal Swamp and surrounding areas. The population of bears is growing and spreading to practically all counties of the Commonwealth. The total number of bears in Virginia may be about 7,000 animals. Bears are nearly always solitary animals, with the exception of family groups (sow with cubs). The male and female have little or no contact other than during mating. Females spend a lot of effort feeding and fattening up before going into hibernation in order to produce healthy cubs. As part of a study conducted by the VA Department of Game and Inland Fisheries and VA Tech, small samples of female black bears or sows were placed into captivity before hibernation. The sows were pregnant and gave birth to their cubs during the winter hibernation. The purpose of the study was to obtain growth data on cubs in order to be able to assess the health of cubs in the wild population. Similar data is being collected on sows and their cubs in the wild for comparison.

The cubs are born at about 224 grams or about the size of a guinea pig; they grow rapidly. At one year, cubs will grow from the 224 grams to about 22.50 to 31.50 kg. As part of the research project, every 10 days, the sows were immobilized and the cubs were weighed and other growth data recorded. Afterward, cubs were returned to their mothers who then woke up and continued to care for them. Once spring arrived and food was again plentiful, the sow and her cubs were released back into the wild.

More information on black bears and Virginia's Bear Management Plan can be found at www.dgif.virginia.gov.

(from Wild about Math, 2005 by Virginia Department of Game and Inland Fisheries)

WEIGHT DATA

Age of Cub (days)	1 day	5 days	45 days	85 days	350
Weight of Cub (gms)	300	581	1940	2400	24450

DATA ANALYSIS

Regression Model

1. Consider a non-linear model for the weight of the Black Bear Cub. Calculate an appropriate Cubic Model for the weight of the bear cubs in the first year of life. Plot the weight – time graph for the first year of the cub's life.

2. What are the limitations of this model for future predictions?

Average Growth Rate at 30 days

3. Calculate the average rate of change in terms of h, for the bear cub's weight between x = 30 and x = 30 + h, where x = day and h is small.

4. Evaluate the limit, $\lim_{h \to 0} \frac{f1(30+h) - f1(30)}{h}$ for the Bear Cub.

5. Check your answer to part 3, by calculation of f1'(x) when x = 30

Further Investigations and Inferences from the Models

6. Using this model, what is the cub's predicted weight at
   a. 30 weeks and
   b. then at 40 weeks

7. What is the average rate of change in the cub's weight from 30 weeks to 40 weeks?

8. Find the gradient of the chord between day 210 [(x) = 210] and day 211 [(x) = 211].

9. Calculate the instantaneous rate of change for the bear cub's weight at 30 weeks and 40 weeks

10. When does the bear cub reach 15 kilograms in weight and at what rate is the bear cub's weight increasing at this time?

11. When is the rate of increase in the weight of the bear cub = 25 grams per day and what is the weight of the cub at this phase of its growth?

12. Does the bear ever lose weight? If so, during what period of time is this occurring? If not, what is the slowest rate of growth for the bear cub?

DATA ANALYSIS SOLUTIONS

Regression Model

1. Consider a non-linear model for the weight of the Black Bear Cub. Calculate an appropriate Cubic Model for the weight of the bear cubs in the first year of life. Plot the weight – time graph for the first year of the cub's life.

2. What are the limitations of this model for future predictions?

1.1 | 1.2 RAD AUTO REAL

	A day	B wgt	C	D	E	F
◆						
1	1	300				
2	5	581				
3	45	1940				
4	85	2400				
5	350	24450				

C1 |

Table 1a: Data Entry

1.1 | 1.2 RAD AUTO REAL

	day	B wgt	C	D	E
◆					=CubicReg(c
2	5	581		RegEqn	a*x^3+b*x…
3	45	1940		a	.001545
4	85	2400		b	⁻.505851
5	350	24450		c	56.8185
6				d	273.973

Table 1b: Cubic Regression

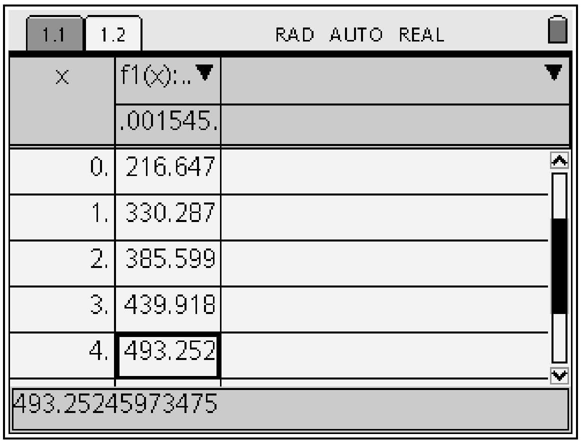


Table 1c: Function Table

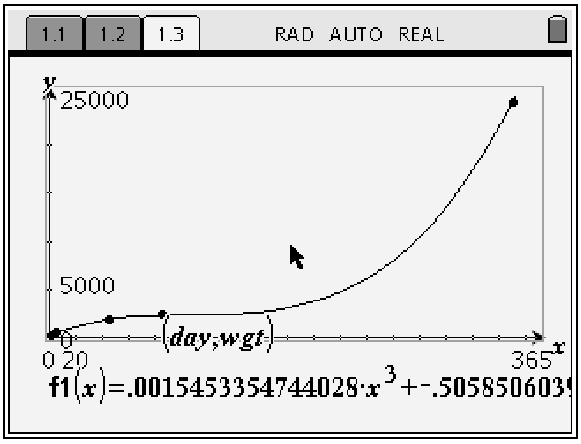
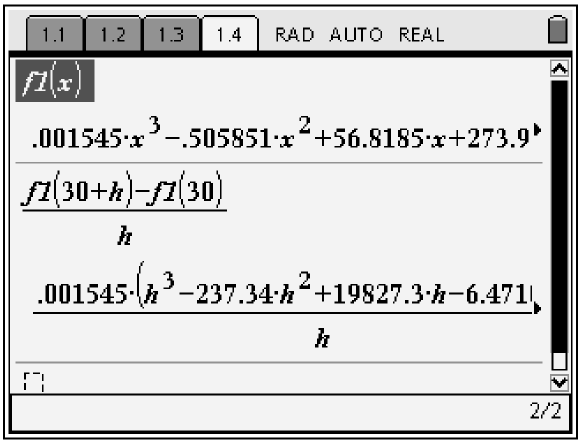


Table 1d: Graph of Cubic Regression

Average Growth Rate at 30 days

3. Calculate the average rate of change in terms of h, for the bear cub's weight between x = 30 and x = 30 + h, where x = day and h is small.



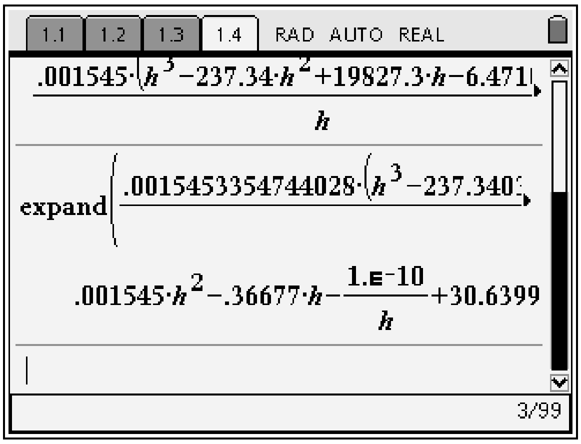


Table 3a

Table 3b

4. Evaluate the limit, $\lim_{h \to 0} \frac{f1(30+h) - f1(30)}{h}$ for the Bear Cub.

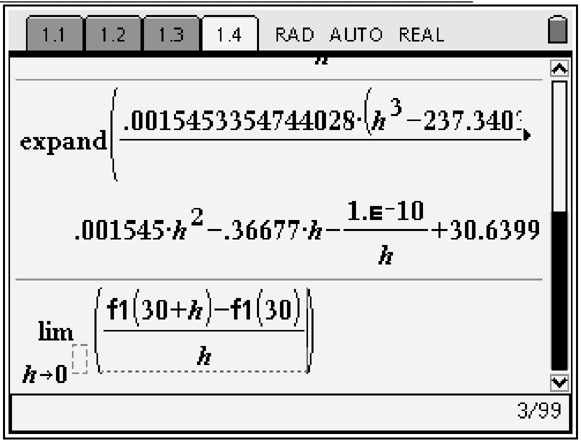


Table 4a

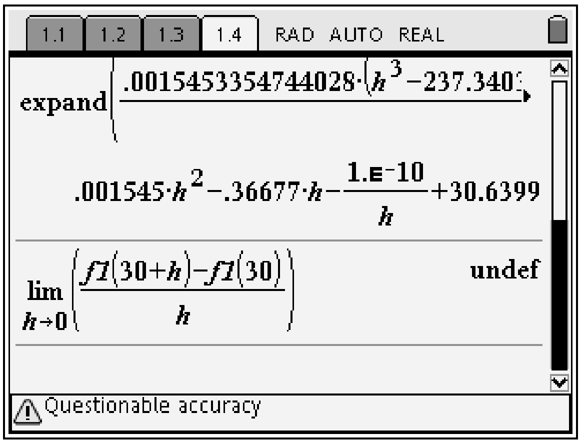


Table 4b

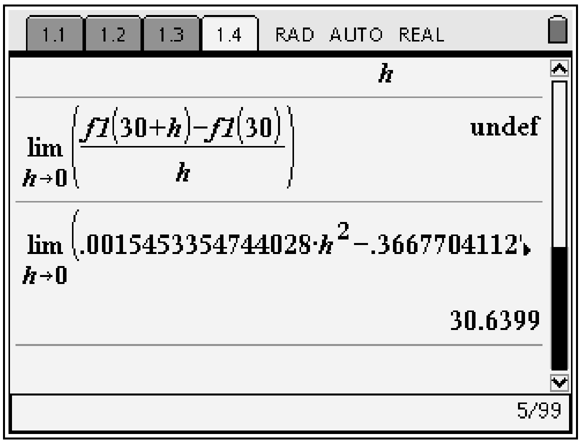


Table 4c

(note: In Table 4b – the rounding procedures in the calculator give a very small value 1E-10/h. In theory this value is zero. It causes the undefined issue, and has to be removed from the limit calculation, as in Table 4c)

5. Check your answer to part 3, by calculation f1'(x) when x = 30

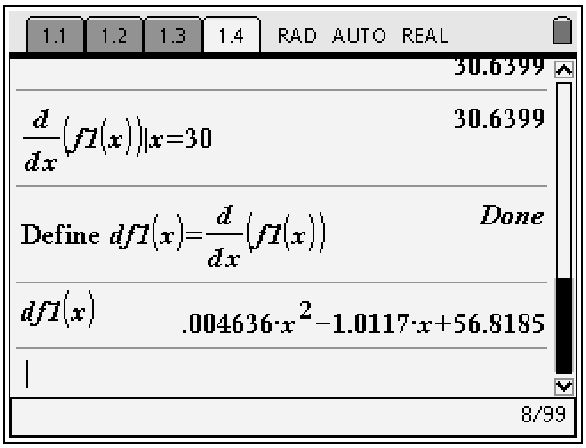


Table 5

Further Investigations and Inferences from the Models

6. Using this model, what is the cub's predicted weight at
    a. 30 weeks and
    b. then at 40 weeks

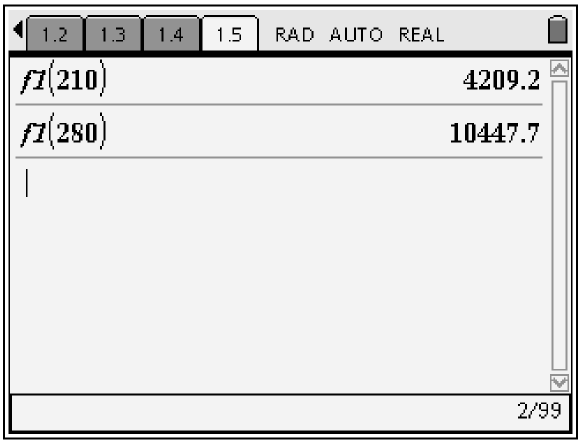


Table 6

7. What is the average rate of change in the cub's weight from 30 weeks to 40 weeks?

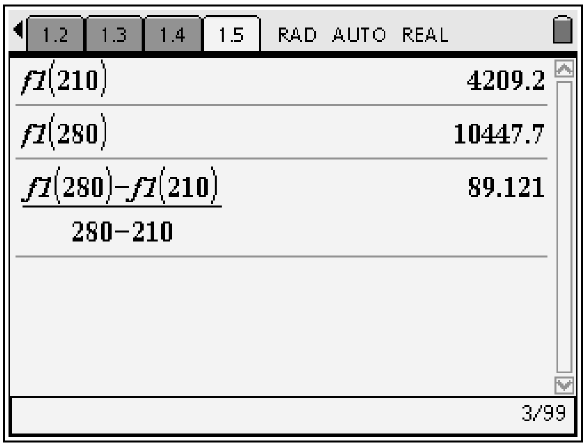


Table 7

8. Find the gradient of the chord between day 210 [(x) = 210] and day 211 [(x) = 211].

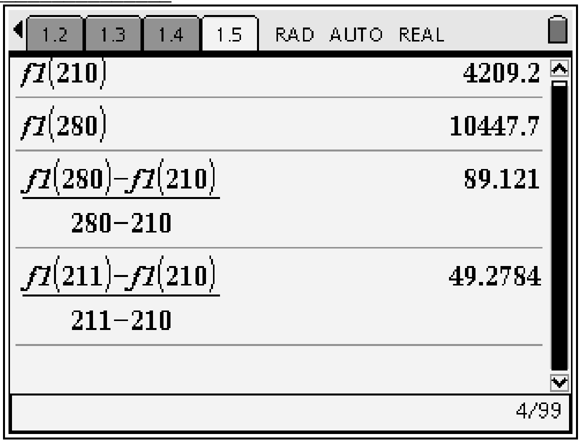


Table 8

9. Calculate the instantaneous rate of change for the bear cub's weight at 30 weeks and 40 weeks

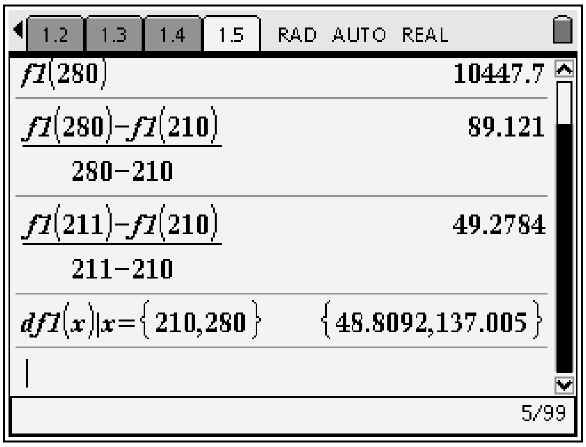


Table 9

10. When does the bear cub reach 15 kilograms in weight and at what rate is the bear cub's weight increasing at this time?

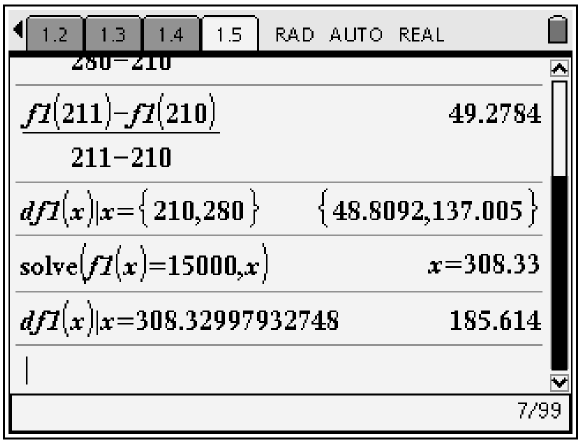


Table 10

11. When is the rate of increase in the weight of the bear cub = 25 grams per day and what is the weight of the cub at this phase of its growth?

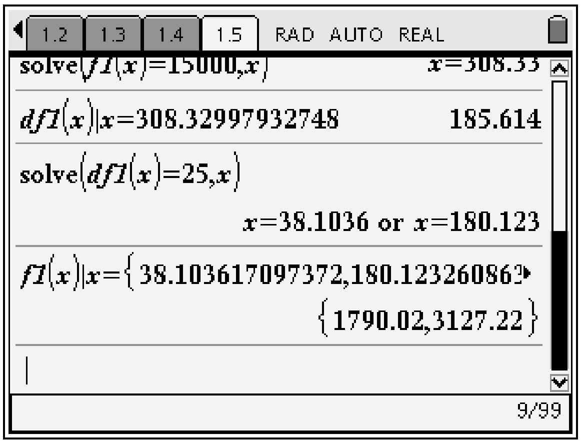


Table 11

12. Does the bear ever lose weight? If so, during what period of time is this occurring?

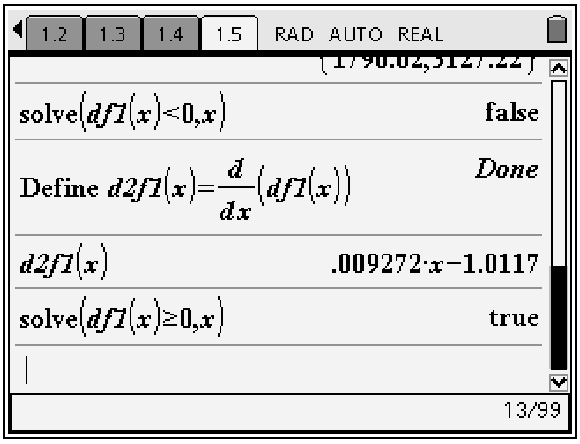


Table 12a

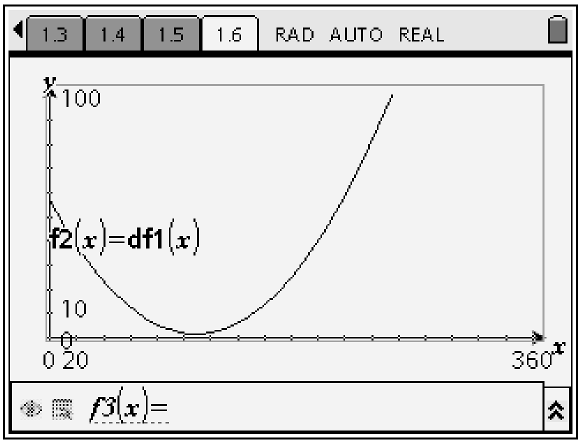


Table 12b

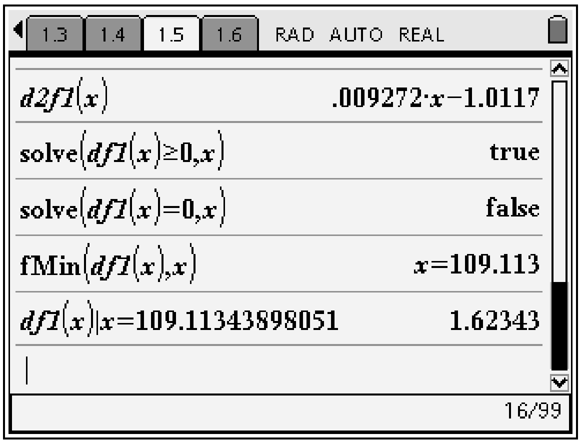


Table 12c