TI-nspire™

The Falling Ladder

By Stephen Arnold

Grade level: secondary (Years 9-12)
Subject: mathematics
Time required: 90 minutes

Activity overview

What does it feel like to be at the top of a ladder as the bottom begins to slide away? Do you fall at a steady rate? If not, then what is the nature of your motion – and when are you falling fastest?

This modelling problem is suitable for students across the secondary school, from consolidation of work on Pythagoras' Theorem in the early years, to optimization using differential calculus in the senior years. At all levels, it is a realistic and valuable task, which links a variety of mathematical skills and understandings with a practical real-world context.

Background

Various forms of this problem have been used for many years, with a range of options available online. None appear, however, to study motion at the *top* of the ladder in the variety of ways approached in this task.

Concepts

Pythagoras' Theorem, Co-ordinate Geometry, Algebraic Modelling, Function Graphing, Optimisation both with and without use of differential calculus.

Teacher preparation

Ideally, at all levels, this task should begin with some experimental work by the students. Initially, this may involve the use of rulers, textbooks, or longer straight-edges. Where possible, motion-detectors may be used to gather real-world data from these experiments. This is a suitable introductory activity for the geometry, graphing and algebraic features of TI-Nspire.

Classroom management tips

This task is best achieved using pairs or small groups working collaboratively, hence the classroom should be able to be easily arranged in pairs or clusters. Depending upon the time wishing to be allocated to this task, the class could be divided into two: one half working on the "overhanging ladder" the other on the "leaning ladder". As an ongoing project, of course, one problem can serve as an extension of the first.

Ideally, students should first be engaged in the task individually for some time (at least 15 minutes). This gives time for some degree of commitment to the task, but not so long that those who are unable to make progress are frustrated. Concrete materials should be available at this stage.

At the next stage, students should be asked to work in pairs, bringing their own understandings and progress to the partnership, and benefiting from the input of their partner. (If additional scaffolding is needed, then teachers may later choose to join pairs into groups of four for further input, discussion and support).

At regular intervals, using a teacher display, groups and individuals should be asked to briefly report on their progress, offering further scaffolding for class members.

This task should culminate in a detailed report on the progress through the various aspects of the task, and so, at regular intervals, students should be asked to clearly summarise their thinking and understandings for each stage of the modelling process.

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Technical prerequisites

Students should know how to:

- Construct basic geometric objects (points, lines, segments, circles…) and use features such as Measurement Transfer, Locus and Calculate. (**Graphs & Geometry** application)
- Transfer Measurements to co-ordinate axes so as to link x- and y-co-ordinates from a geometrical construction to a function graphing environment. (**Graphs & Geometry** application)

  (NOTE: the geometric constructions are NOT required for this task unless the teacher specifically wishes students to create their own constructions. Generally, students would use the "ladder tools" provided with which to explore the problem).
- Store values from geometric constructions as variables, which can then be used in algebraic constructions. (**Lists & Spreadsheet** application, **Calculator** application and **Graphs & Geometry** application)
- Define lists and use Automatics Data Capture to transfer dynamic measurements to lists (**Lists & Spreadsheet** application and **Graphs & Geometry** application).
- Create a Scatter Plot from defined list variables (**Lists and Spreadsheet** application and **Graphs & Geometry** application).
- Define and graph functions which can then be used to graphically verify an algebraic model against a geometric locus. (**Calculator** application and **Graphs & Geometry** application)

- OPTIONAL: Use a CBR 2™ or other appropriate motion-detector to collect physical data from their initial experiments.

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Step-by-step directions

STAGE 1: Setting the context of the task, and early (physical) investigation. 30 minutes

1. Briefly present the task, using the three focus questions provided: (i) What does it feel like at the top of a ladder, as the bottom begins to slide away; (ii) If the bottom slides away at a steady rate, do you also fall at a steady rate? (iii) If not, then what is the nature of your motion, and at what point are you falling the fastest?

2. Explain the nature of mathematical modelling, and the various tools available from which they will be required to construct a variety of models: physical, geometric, graphical and finally, algebraic. Observe that there are two possible scenarios for this problem: the ladder leaning against a wall, and the ladder overhanging the wall. Both should be considered.

3. Invite students to begin their investigation individually and then, after around 15 minutes, to join with a partner and compare ideas and progress.

4. *Pairs should be invited to briefly describe their understandings of the task. They may also be invited to enter their comments into the NOTES application of TI-Nspire*

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1. Students are invited to continue their investigation using the "ladder tools" provided on TI-Nspire. Some brief discussion of the construction of the tools may be desired – note, especially, the important ideas of **variable** and **domain** implicit in the construction.

2. Working in pairs now, the class may be divided in half: some students working with the "overhang" problem, others with the "leaning" problem, all attempting to draw from the geometric model a clear description of what is happening.

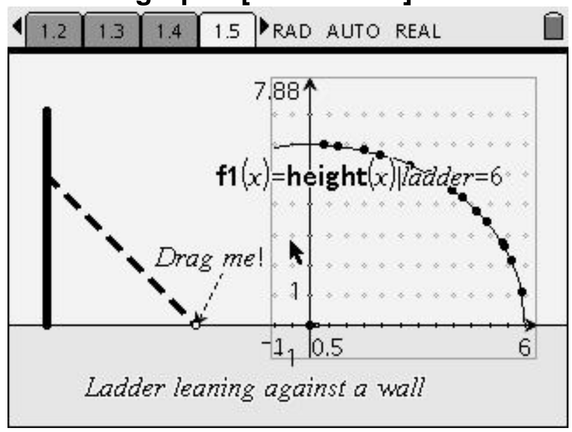
3. The geometric models have been set up to support **Manual Data Capture**: moving the variable point to different positions and pressing **CTRL-(.)** (period) will drop a new point on the graph, building up a graphical model of the motion.

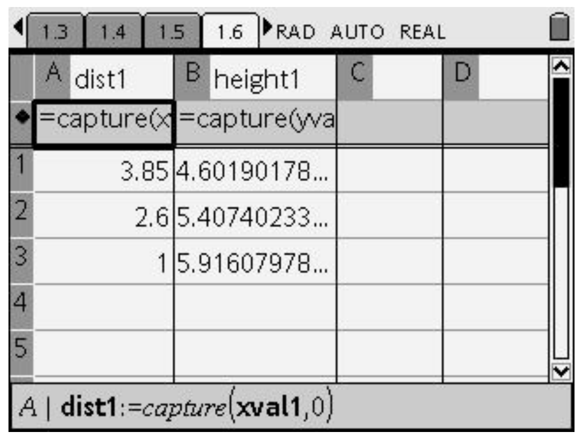
4. Attention might be drawn to the table of values provided by the **manual data capture** facility, and then back to the co-ordinate axes, where the **Scatter Plot** has been constructed.

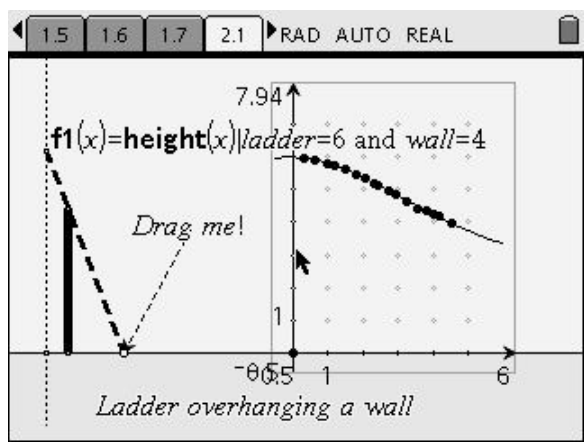
5. If another, simpler approach is desired, then the Locus tool may be introduced (for those who have not already discovered it!)

6. *Again, groups of students should briefly report back to the class, comparing and contrasting their approaches and conclusions.*

STAGE 2: Geometric investigation: From geometric model to numerical model; from number to graph. [30 minutes]







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STAGE 3: Algebraic Modelling Phase
30 minutes

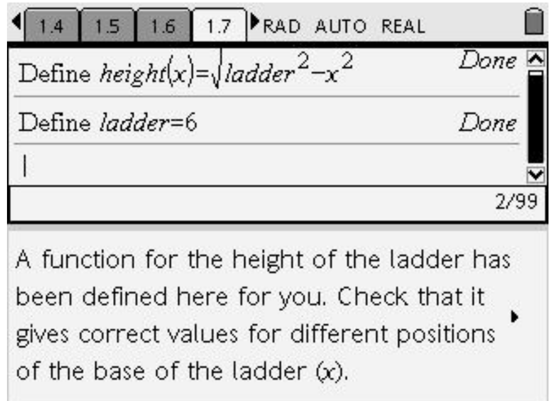
1. Using variables allocated to the different components of the geometric model, students attempt to build an algebraic model, which satisfies the constraints of the task.

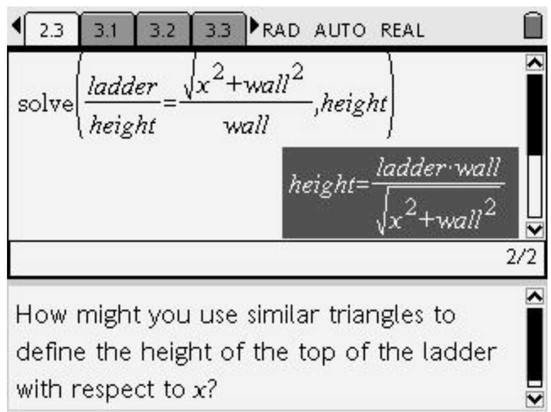
2. Graphing the function associated with their algebraic form allows them to verify the accuracy of their model.

3. This model may then be used to compute the answers to set questions: (i) At what value of *x* does the top of the ladder reach the top of the wall; (ii) where is the top of the ladder when *x* = 0? (iii) At what value of *x* are you falling the fastest? Justify your answer.

4. The concluding phases of the activity should involve discussion and debate regarding the limitations of the models chosen, including calculations of values when one model takes over from the other.

5. Students should prepare a detailed report of their investigation which includes key values (including domain of ***x***), graphical and algebraic solutions to the task, and strengths and weaknesses of their models.





(Possible CAS extension)

Assessment and evaluation

This task may be assessed in a variety of ways, depending on the year group for whom it is intended. For younger students, the focus should be upon the application of Pythagoras' Theorem to building an algebraic model of each situation; senior students should attempt the task as an optimization problem: at what point are you falling the fastest?

In all cases, there should be a clear focus upon the modelling process: using a range of approaches leading to a correct algebraic model, which supports both clear description and potential prediction. The students need to be able to put what is happening into their own words, and to interpret the various representations (geometric, numeric, graphical and algebraic) in a meaningful way.

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THE FALLING LADDER PROBLEM

If ***x*** represents the distance of the foot of a ladder of length 6 metres from the base of a 4 m wall:

1. What is the height of the top of the ladder above the ground when ***x* = 0**?

2. If the ladder is leaning against the wall, then find the height of the ladder above the ground when ***x* = 5 metres**.

3. If the ladder is *leaning on* a wall of height 4 metres, then what is the length of the overhang when ***x* = 2 metres.**

4. Find the value of ***x*** when the overhang is 0?

5. What function best describes the relationship between the distance from the base of the wall (***x***) and the height of the ladder leaning against the wall?

6. What function best describes the relationship between **x** and the *overhang* of a ladder of length ***ladder*** and a wall of height ***wall***?

7. What function best describes the relationship between **x** and the height of the top of a ladder overhanging a wall?

8. At what (exact) value of ***x*** does model *f1* take over from *f3*?

Answers:

1. When *x* = 0, the height of the ladder above the ground is 6 metres (the length of the ladder).

2. When *x* = 5 metres, the height of the ladder against the wall would be approximately **3.3 metres.**

3. The overhang when *x* = 2 metres is approximately **1.5 metres**.

4. When the overhang is 0, then *x* is approximately 4.5 metres.

5. $f1(x) = \sqrt{ladder^2 - x^2}$

6. $f2(x) = ladder - \sqrt{x^2 + wall^2}$

7. $f3(x) = \dfrac{ladder \times wall}{\sqrt{x^2 + wall^2}}$

8. $x = 2\sqrt{5}$ (approximately **4.5**)
[When *f3(x)* = h, with l = 6, h = 4)

These questions offer only suggestions. Students may need more or less scaffolding, depending on their familiarity with the modelling process, and their prior experience with extended investigative tasks.

A “golden rule” for teachers: *tell them as little as possible*! Avoid the temptation to give too much information and encourage them to persist and to engage actively with the task.

Encourage, too, the use of the technology to create an extended response which makes full use of the variety of representations available here.

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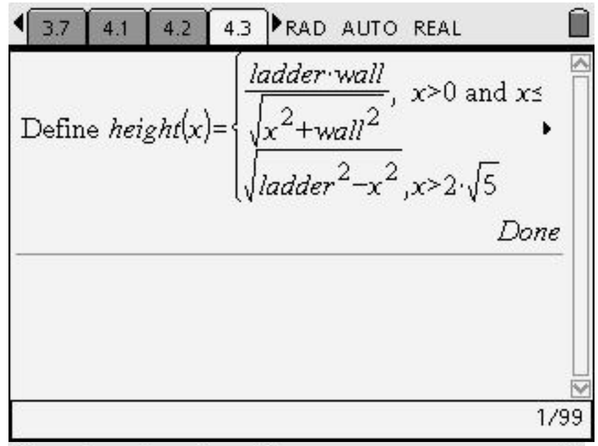
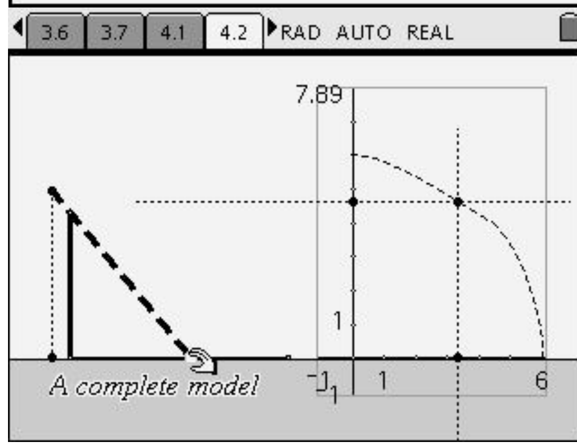
Activity extensions

Extension 1:

Put it all together to build a function which models the entire process – from the ladder overhanging the wall which becomes the ladder sliding down the wall after passing the top of the wall!

NOTE that you fall fastest just before you hit the ground (some argue that you fall with infinite speed – can you see why? Can you explain why not?)





Extension 2:

What if the wall was NOT perpendicular to the ground?