Meeting with a Friend

By: Stephen Arnold

Grade level: secondary (Years 9-12)
Subject: mathematics
Time required: 90 minutes

Activity overview

My friend and I agree to meet during our lunch hour. If we each decide to wait for 15 minutes, what is the probability that we will meet? How long should we agree to wait in order to have a 50% chance of meeting? How long for an 80% chance?

Background

A version of this problem was set as the final question for the 2005 New South Wales Higher School Certificate examination in Mathematics. Copyright is held by the New South Wales Board of Studies.

Concepts

Probability, linear inequalities, algebraic modeling

Teacher preparation

Prior to attempting this activity, students should be familiar with linear inequalities and their graphs. They should also have encountered key concepts of probability.

Classroom management tips

This problem is best set as a collaborative problem-solving task, which students should attempt in pairs or small groups using the technological tools supplied. Regular breaks when students share their thinking and progress with the class would serve a useful scaffolding function.

All students should be required to keep clear and detailed records of their progress through this problem, their observations and questions.

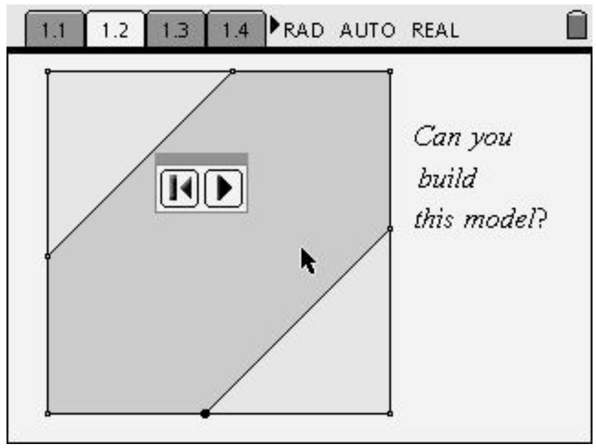
Technical prerequisites

Students should know how to:

- Construct a square using axes;
- Reflect points and shapes in lines;
- Measure length and area of geometric objects;
- Transfer measurements from geometric figure to axes for graphing of relationships.

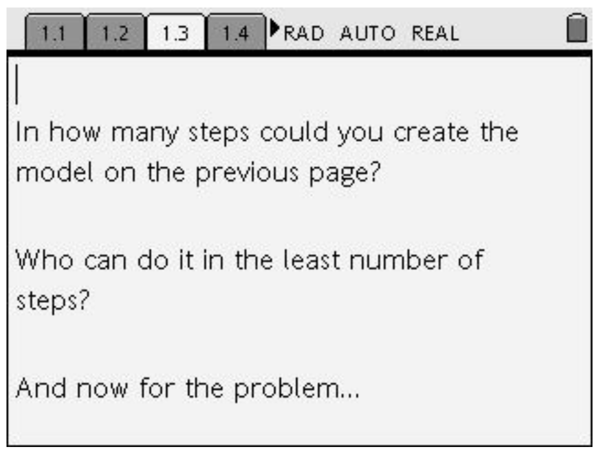
Step-by-step directions

1. Depending on the level of ability and experience of the class, they may either be expected to build their own geometric model for this task, or be provided with one (as shown). Ideally, the teacher would demonstrate the model by dragging the variable point along the base, and allow students to observe the effect. Students may then be challenged to build their own version of this.

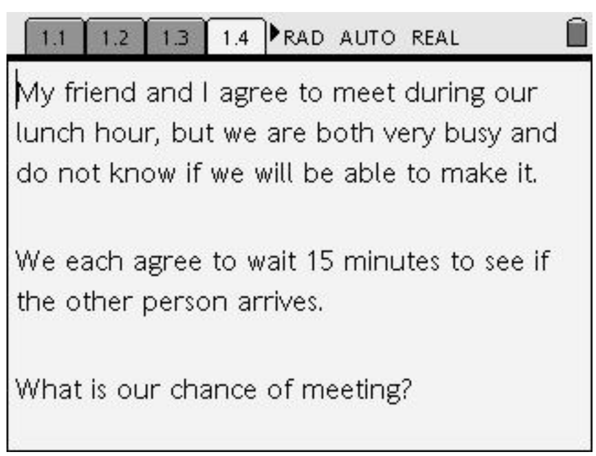


2. The creation of the geometric model may be set as an independent (prior) activity to the setting of the task question concerning the meeting of friends.

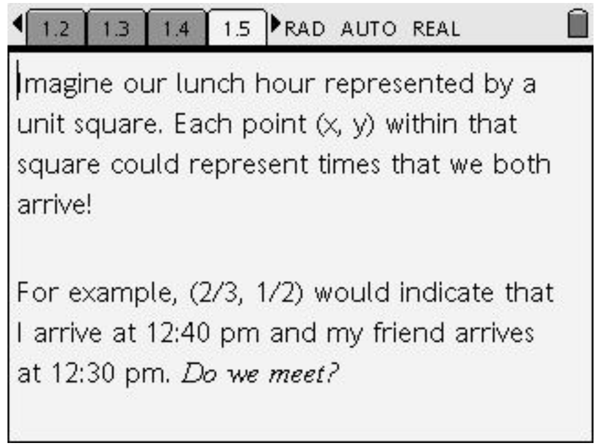
   Students should be given time to explore this problem in their own ways before introducing the particular approach offered here. Some may make the connection with the original geometric model; most probably will not.



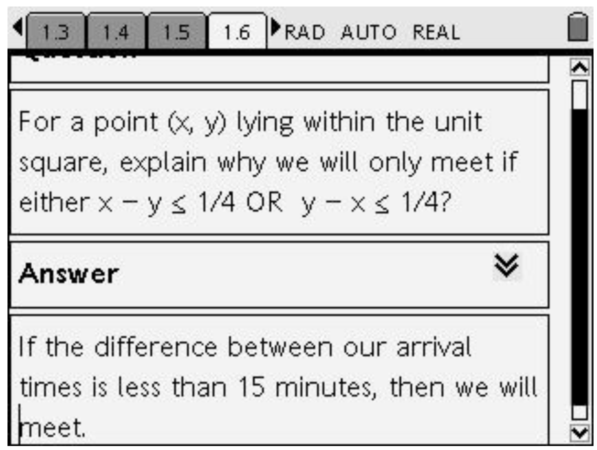
   Much may be gained by the teacher collecting from students their estimates concerning the probabilities associated with various waiting times, and this data may be entered into lists and graphed, or an intuitive graphical model may be explored. This should take the form, initially, of discussing the suitability of **y = x** as a model for this situation, and then adjusting this to better suit what students already know (for example, what do you know for very short waiting times and very long waiting time?)



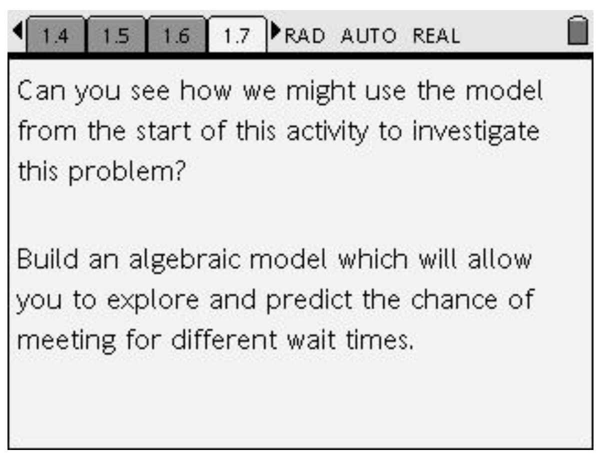
3. The geometric model may then be introduced carefully by the teacher, inviting discussion from the class. The use of fractions to represent various arrival times will need some thought on the part of students, who may be unfamiliar with such a concept.



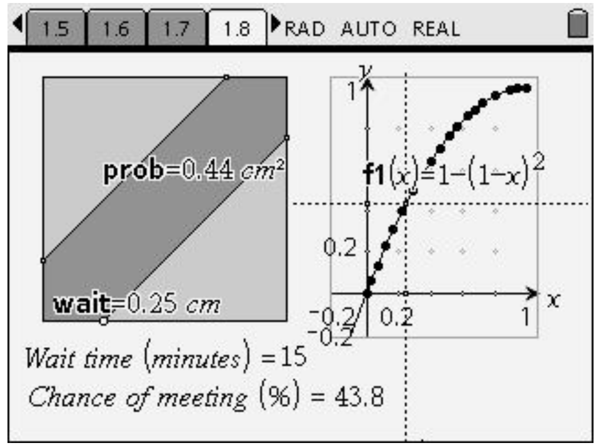
4. The introduction of algebraic inequalities may not come naturally to students, and time will be needed for them to explore the graphs of these and observe the way in which they fit the situation. The early introduction of the interactive geometric model is likely to help significantly in students making meaning of this representation, and should be referred to when necessary and appropriate.



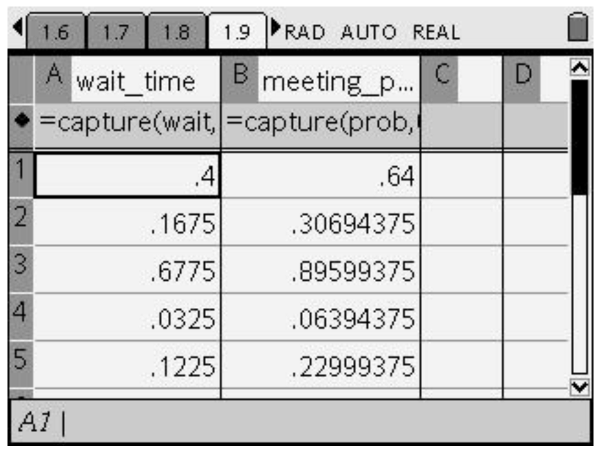
5. Finally, introducing the geometric model once more, with the x-variable and the area of the hexagon shown will invite students to draw further connections between the problem situation and the geometric model.



6. The final stage of the task lies in students building an algebraic model of the relationship between the value of **x** and the area of the hexagon as representing the probability of the two friends meeting. This should begin experimentally, with students linking the data from the geometric figure to a graphical locus, which then becomes the basis for verifying their conjectured algebraic model.



The importance of this experimental approach, culminating in graphical verification of the algebraic model, should not be under-estimated. It offers a powerful means for students to build meaningful connections between the various representations, and to appreciate the value and importance of algebraic methods as precise means of being able to predict actual real-world implications.



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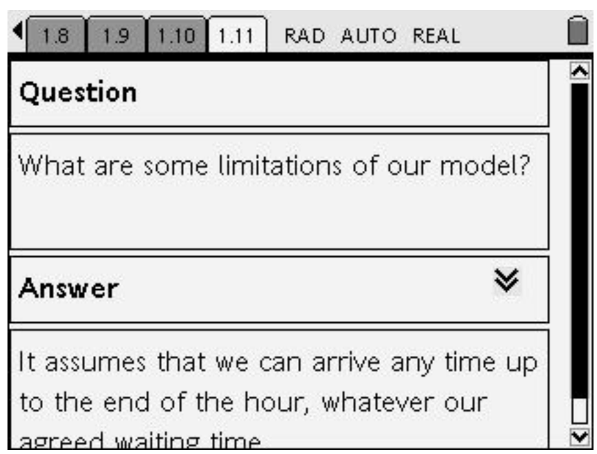
Of course, any modeling experience should always involve some critical evaluation of the strengths and limitations of the model!



Assessment and evaluation

Assessment for this activity should build from the particular to the general, as students initially use the model to answer simple questions (and so draw connections between the real-world situation and the various mathematical representations used here), and gradually work towards an algebraic solution which matches their numeric results, but offers the potential for both greater precision, and for prediction.

1. If the two friends agree to wait for 15 minutes, what is their chance of meeting?	43.75% (approximately 44%)
2. How long should they wait for the probability of meeting to be 50%?	17.6 minutes
3. For a probability of 80%?	33.12 minutes.
4. Explain why the two friends will only meet if either **x – y <= 1/4 OR y – x <= 1/4**.	The key here lies in the difference: as long as the difference between arrival times is less than one quarter of an hour (for the original question) then the two will meet. The two versions? Depending on who arrives first!
5. What algebraic model will satisfy these conditions, for a unit square?	**$P(x) = 1 - (1 - x)^2$**

Student reports of this investigation should be clear and detailed; observations and clear explanations of each representation (physical, geometric, graphical and algebraic) should be required. As always, for algebraic modeling, there should be clear recognition of the assumptions made and the limitations of the modeling process.

Activity extensions

- How might this problem be generalized for longer and shorter periods of time? For example, if the lunch break was only 45 minutes, or there was a two-hour period available, in what ways will this affect the results, and the validity of the model.

- What are the strengths and limitations of this modeling process – how might these limitations be minimized?

