TI-nspire™

Exploring the Parabola

By Stephen Arnold
Grade level: secondary (Years 7-12)
Subject: mathematics
Time required: 90 minutes

Activity overview

This activity explores the key features of the parabola, both geometrically and algebraically. A variety of interactive representations support student learning as they build their understanding of this important curve and its real world applications.

Background

The primary objective in the study of parabolas in many high school curricula, tend to be algebraic, moving quickly to the study of the quadratic function. Key defining features of this function are geometric in nature. Students often misrepresent other curves as ‘parabolic’ simply because they have a similar appearance; the catenary is a classic example (A chain suspended from its ends forms a catenary curve - the word *catenary* is derived from the Latin word for chain). It is therefore important for students to understand some of the properties of a parabola, features that make this curve both unique and important. This activity supports students in actively linking some of the geometric and algebraic properties of a parabola.

Concepts

Locus (especially perpendicular bisector), Parabola and the Quadratic function.

Teacher preparation

Prior to this activity, it is recommended that teachers take some time to build an understanding of locus concepts with their class. This might take the form of physical involvement on the part of the students, as they build from a simple locus to the more complex. A simple locus for students to ‘construct’ is the circle. A volunteer from the class becomes the most popular person in the school – everyone wants to be close to this person. How would you arrange yourselves to all be as close as each other? A more complex situation is the ‘construction’ of a perpendicular bisector. Two volunteers are now equally popular but very jealous – everyone wants to be close to them, but never closer to one than the other! How could the friends of these two people arrange themselves? To model a locus to form a parabola, consider one very popular person, and the most popular group, lined along the wall at the school dance, hand in hand. How will others arrange themselves so as to be equally close to both the individual any single person from the group?
Note: In this last example, due care should be taken to see the ‘group’ as a continuous identity. The linking of hands helps to create this view.

Classroom management tips

This interactive exploration moves between representations, from physical involvement in the building of loci, to geometric construction, to algebraic functions and their graphs. The early work involving popularity contests builds both interest, a sense of fun, and a deeper intuitive understanding of key concepts such as perpendicular bisector.

It is always useful to have students work in pairs, both to encourage verbalization and discussion, and to offer scaffolding when needed. Individuals may work with their own electronic device, or they may work in pairs. Each must keep a record of the process – their own thinking and observations, and questions that arise. Bringing the class back together for sharing and discussion is very important, particularly to keep some from falling too far behind.

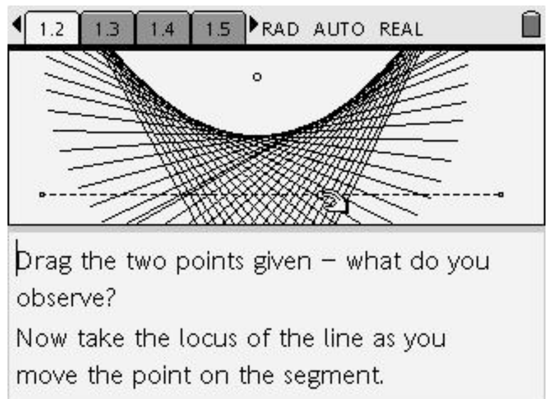
Technical prerequisites

Students should know how to:

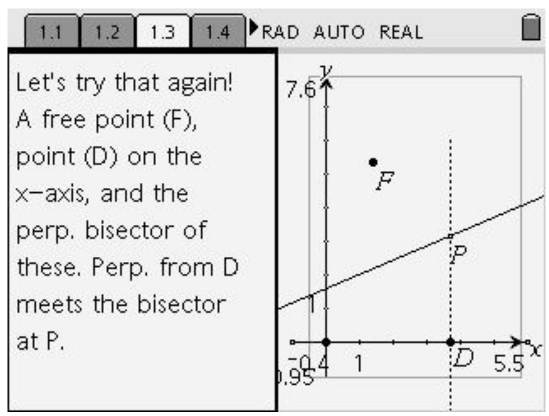
- Construct simple segment and point models, as well as perpendiculars and perpendicular bisectors, locus and reflections.

Step-by-step directions

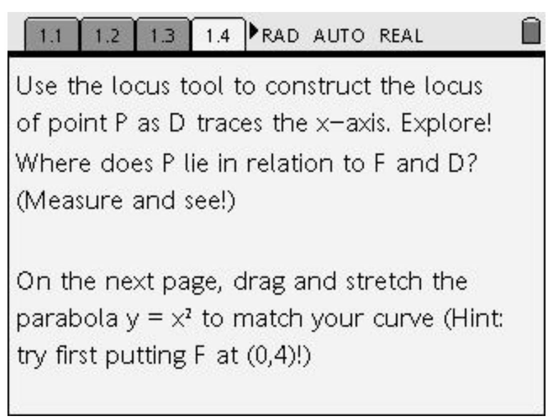
1. This activity begins dramatically as students follow very simple instructions which lead to the envelope shown here.
A nice complement to the use of technology at this point involves some simple paper folding. Taking a sheet of paper, students clearly mark a point somewhere around the centre of the page. They produce a series of folds from along one of the long edges through this point. Tracing along these straight-line folds should produce an envelope similar to the one shown. It is important that students see the connection between the crease in the paper and the perpendicular bisector.



2. The page-by-page instructions provided in the interactive document for this activity allow students to navigate through the procedures themselves. Nonetheless, it is still recommended that the teacher accompany them on this journey, ideally using a viewscreen projection, which allows for class discussion and for teacher comments.



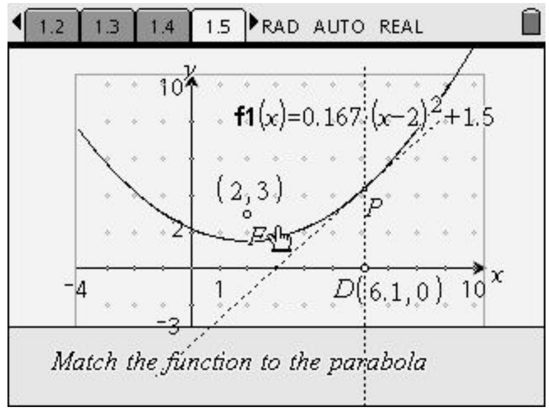
3. The early "play" is important and students should be encouraged to explore the effects of different parts of the construction: move the point, (focus) and the segment (directrix) and observe their role in the locus construction.

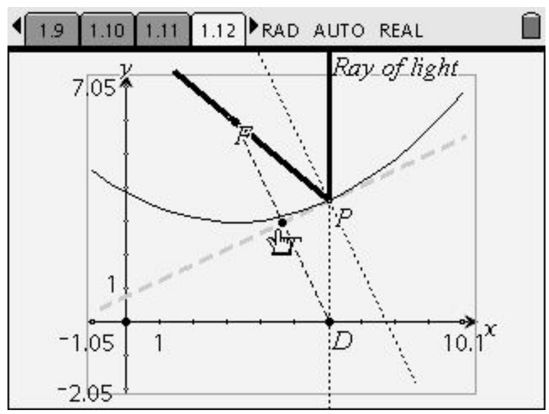


4. Formalising the construction involves labeling key points and some discussion as to why the point **P** has been chosen. For senior and more able junior students, it may be observed that the perpendicular bisector now becomes a tangent to the parabola at **P**.

5. The next phase of the lesson introduces the algebraic notation for the quadratic function, and invites students to engage in the unique “manual regression” which this technology makes possible! As students ‘drag’ (translate) the fundamental curve $y = x^2$ to match the envelope, it must be highlighted that this process simply acts to model the curve, otherwise students may fall into the same trap as Galileo, misrepresenting a suspended chain (catenary) as a parabola.

   Students should be encouraged to explore the relationship between the various components of the algebraic form and the physical features of the curve such as that modeled in the original parabola locus by the students. To build an algebraic understanding, students can use algebra to determine an ‘equation’ for the curve, based on these properties. This amounts to using the formula for the distance between two points.





Finally, a link to the real world – the defining property of the parabola, which explains so many of the applications of this important curve!

Again, students should be given the time and necessary direction to discover and to realize the implications of this feature of our curve. Physical modeling may again play a role: for example, rolling a ball against a sheet of card bent into a parabolic shape.

Assessment and evaluation

- Primary assessment for this activity should be a detailed report of the investigation, with student comments and observations related to the physical, geometric and algebraic properties of the parabola.

- Specific questions could focus on linking the algebraic to the graphical forms:
  1. What is the equation of the parabola with focus at (0,4) and directrix on the x-axis?

     $Y = 0.125\ x^2 + 2$
  2. Find the equation of the parabola with focus at (2,3) and directrix **y = 0**.

     $Y = 0.167\ (x – 2)^2 + 1.5$
  3. Find the focal point for the parabola with equation $\mathbf{y = 0.13\ (x – 3)^2 + 2}$?

     (3, 4)

These questions, of course, lead readily to two more:

1. what is the significance of the co-efficient of **x** in all of these equations, and

   The co-efficient of the x-term dictates the "speed" of the curve – how quickly it increases, and is given by 1/(4f), where f is the focal length (half the distance between the focus and the directrix).
2. Can you find the general form for the equation of a parabola with focus at (a, b) and directrix on the x-axis?

$$y = \frac{1}{2b}(x - a)^2 + \frac{b}{2}$$

Activity extensions

Extension activities should include:

- The derivation of the general form from focus and directrix (given above);
- Calculus applications involving tangent, and where the tangent cuts the x-axis;
- The extension of this construction to other conics (if point **T** is positioned closer to **F** then the curve is an ellipse; closer to **D** produces an hyperbola):