Learning Algebra Using a Variety of Tools

By: Tom Steinke
Grade level: secondary (Years 7-10)
Subject: mathematics
Time required: 90 minutes

Activity overview

This activity explores the equivalence of algebraic expressions in expanded and factored form, using patterning with CAS to expose commonly held student misconceptions.

Background

Increasingly, in curriculum documents in Australia and overseas, you will find expectations that refer explicitly to computer algebra systems (CAS). When computer algebra (CAS) systems are referenced, it is frequently in the context of "using a variety of tools", where the three tools most frequently referenced are algebra tiles, computer algebra systems, and paper and pencil. From our lived experience with previous curricula, it is safe to assume that we all have:

- an excellent sense of what learning algebra with paper and pencil looks like,
- a reasonable sense of what learning algebra with algebra tiles looks like, and
- a minimal sense of what learning algebra with computer algebra systems looks like.

Shifting focus to our students, it is clear that all our secondary students learn algebra effortlessly using paper and pencil – NOT! Next to fractions in elementary mathematics, algebra is probably the most commonly referenced area of math that adults refer to when they talk about where math ceased to make sense to them. So … here we are in a society that expects ALL students to learn algebra. Is this possible using only paper and pencil? The recently deceased mathematics education researcher, James Kaput, who championed the notion of the democratization of mathematics wrote:

> "Now we must find ways to make major mathematical ideas learnable by the large majority of people, infinitely more diverse than the geniuses who originally built them. Fortunately we are no longer constrained by static, inert media - unless we choose to be."
>
> *http://www.simcalc.umassd.edu/downloads/KaputAthens.pdf*

Current curricula worldwide challenge us to support student learning in algebra "using a variety of tools". Computer algebra systems (CAS), algebra tiles, and paper and pencil are all necessary thinking tools to uncover meaning in algebra for all our students. So what does learning algebra "using a variety of tools" look like in a real classroom?

This activity is designed to help students and teachers develop an image of what learning algebra using computer algebra systems looks like in a grade 9 or 10 mathematics classroom. The goal of the activity is to have students use CAS as a thinking tool to uncover meaning and misconceptions in algebra.

Concepts

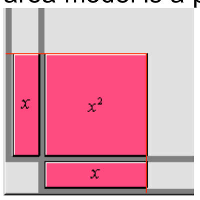
The big algebraic mathematical ideas this activity explore are equivalence and symbol sense. More precisely, the activity speaks to the following curriculum expectations:

- expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials [e.g., $(2x + 3)(x + 4)$] or the square of a binomial [e.g., $(x + 3)^2$], using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil) and strategies (e.g. patterning)

Teacher preparation

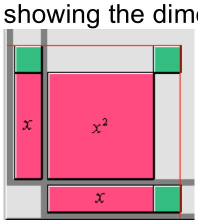
It would be powerful to spend a class exploring area models for multiplication using algebra tiles.

Asking what x*x is, is equivalent to asking what the area of a square with side length x is. The use of an area model is a powerful visual model to represent polynomial multiplication.

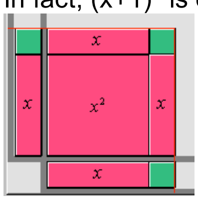


Screen captures are from The National Library of Virtual Manipulatives:
Go to *http://nlvm.usu.edu/en/nav/topic_t_2.html* and choose the first option: **Algebra Tiles**.

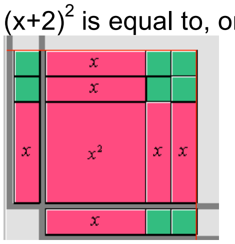
Why $(x+1)^2$ is NOT equal to, or equivalent to $x^2 + 1$?
It is a common misconception that $(x+1)^2$ is equal to, or equivalent to $x^2 + 1$. It is clear from the algebra tile area model, that this cannot be true. Each green tile represents 1. The side algebra tiles are guide tiles showing the dimensions of the area model.



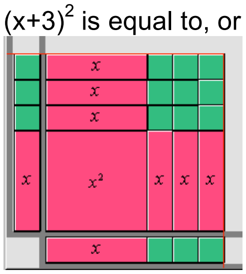
In fact, $(x+1)^2$ is equal to, or equivalent to x^2 + **2x** + 1.



$(x+2)^2$ is equal to, or equivalent to $x^2 + 4x + 4$.



$(x+3)^2$ is equal to, or equivalent to $x^2 + 6x + 9$.



To prepare for the actual TI-Nspire lesson, the teacher should load the class file on the handhelds.

Classroom management tips

It would be desirable to have students working in pairs to enable dialogue. It is important that the teacher actively circulate around the class observing what students are saying, thinking and writing. It is important for the teachers to allow students to work through their misconceptions with CAS and their partners.

If this is the first time students are using TI-Nspire, it would be worthwhile to spend some time getting the students comfortable with the tool. The activities are designed to be accessible to any user of TI-Nspire – new or experienced.

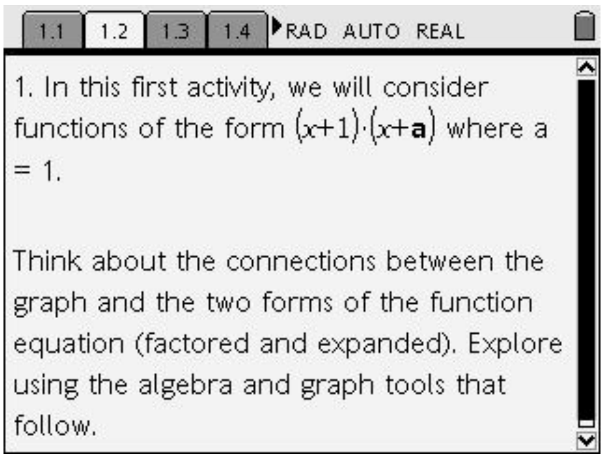
Technical prerequisites

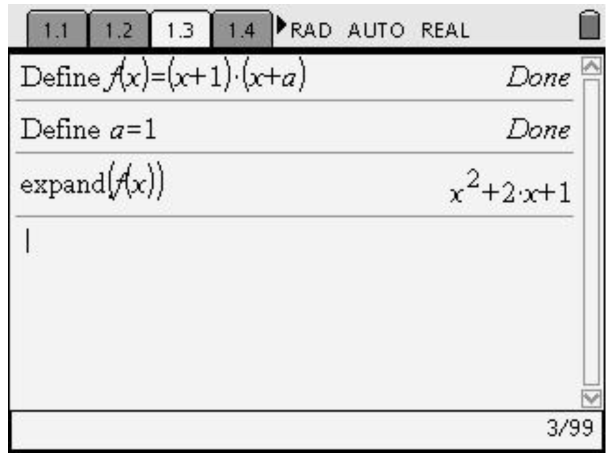
Students should know how to:

- Understanding of “define” and “expand” commands in the calculator environment

Step-by-step directions

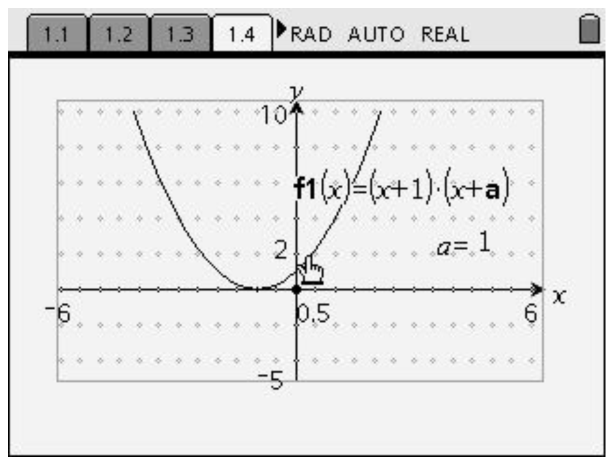
1. The first stage requires students to define the function as shown, define a starting value for "a" and then **expand** in the **Calculator** application.



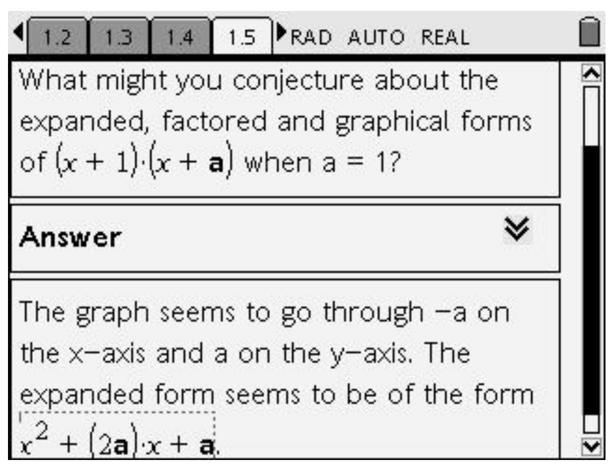


At this point, students should record their observations around how they believe the following are connected:

- the factored form of the function equation,
- the standard form of the function equation, and
- the graph of the function, including the x-intercept(s).



It should be noted, that many students have a persistent belief that $(x+1)^2$ is equivalent to x^2+1. This misconception may have been clarified with the earlier algebra tile activity. If the misconception still persists, the CAS suggested equivalence of $(x+1)^2$ and x^2+2x+1 will cause these students to pause and reflect.

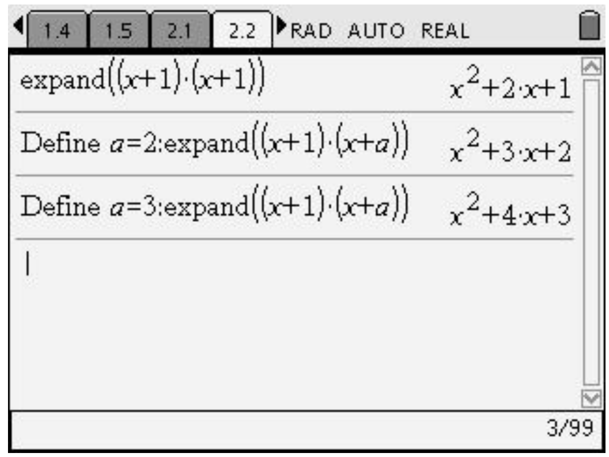


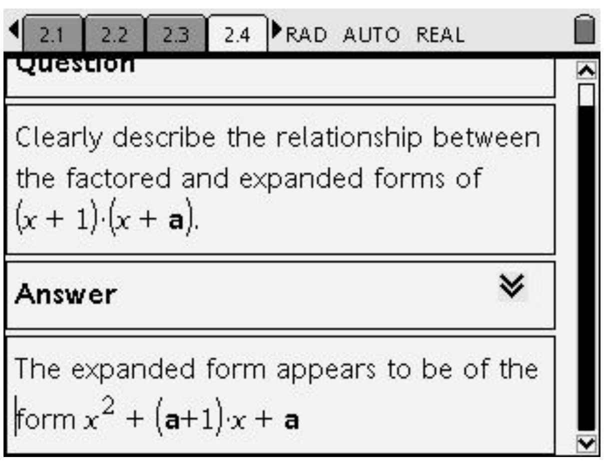
2. The next stage requires students to investigate the effect of other values for “a” in the function.

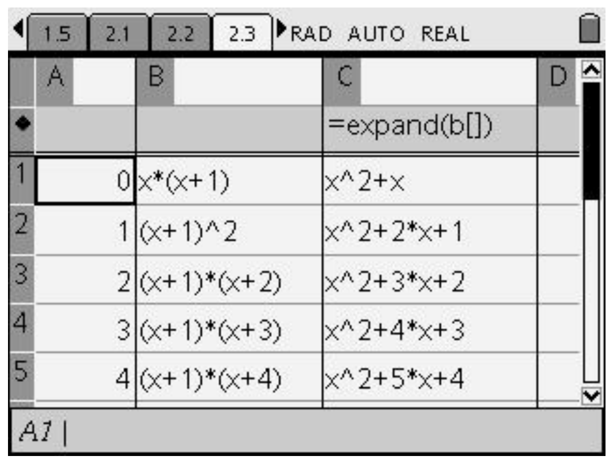
   A pattern-building approach should be encouraged. This may involve using the Calculator, as shown, but also the “Fill Down” property of the **Lists & Spreadsheet** application.

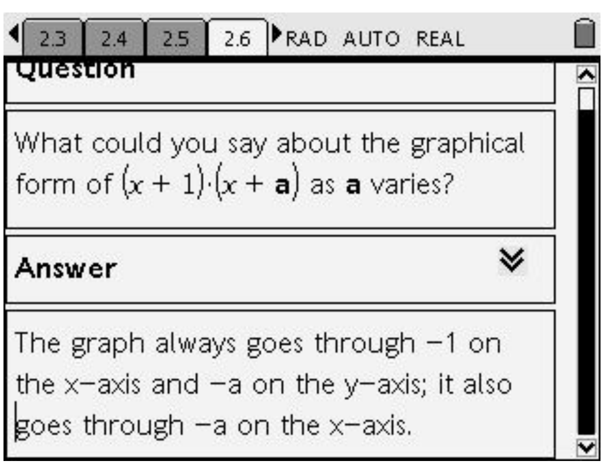
   Students should attend to the equivalence of (x+1)(x+2) and x^2+3x+2 (and subsequent forms) in their observations.

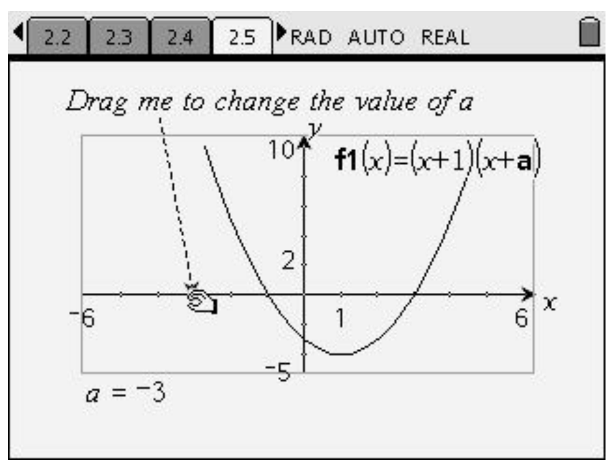
   Students should be asked to make and check predictions as to the algebraic and graphical forms generated.





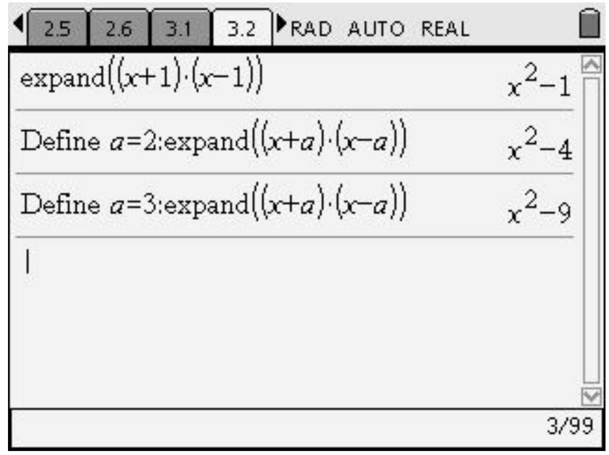


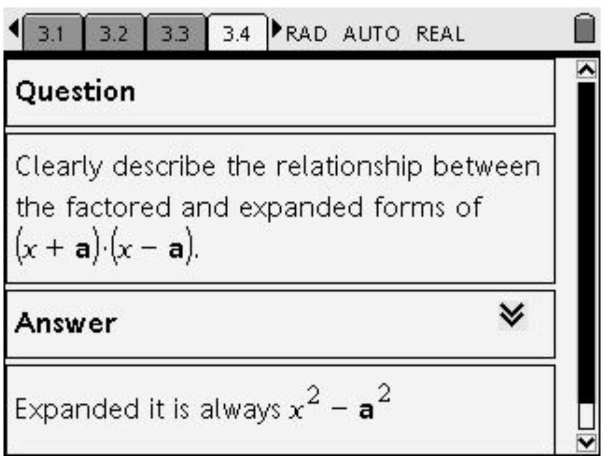


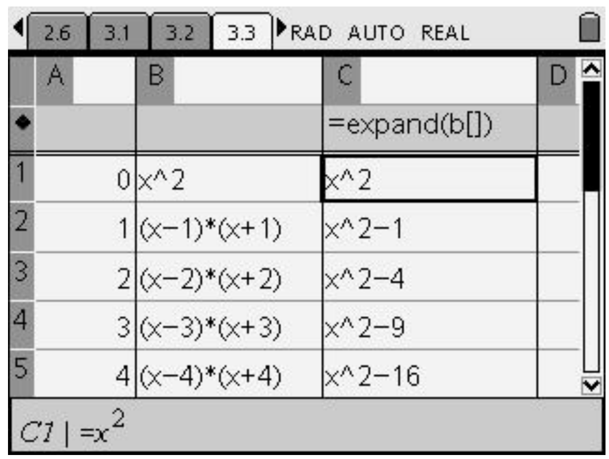


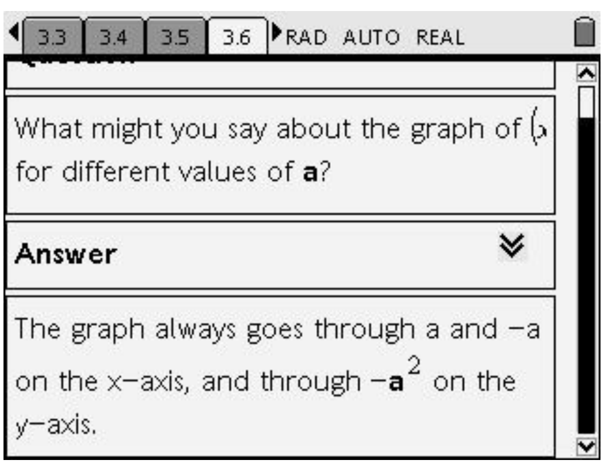
3. In the next stage, we consider the case of the **difference of two squares**.

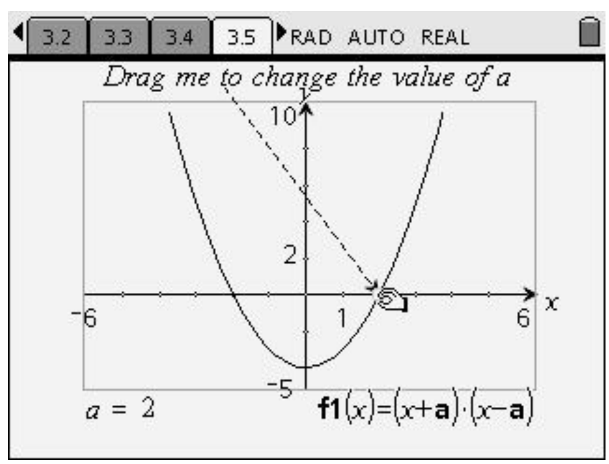
   In this exploration of difference of squares, students may have developed an expectation that the standard form of a quadratic expression must have three terms. Students may be looking for the very middle term they didn't believe existed in the equivalence of $(x+1)^2$ and x^2+2x+1. This potential dissonance will be a rich opportunity for dialogue.





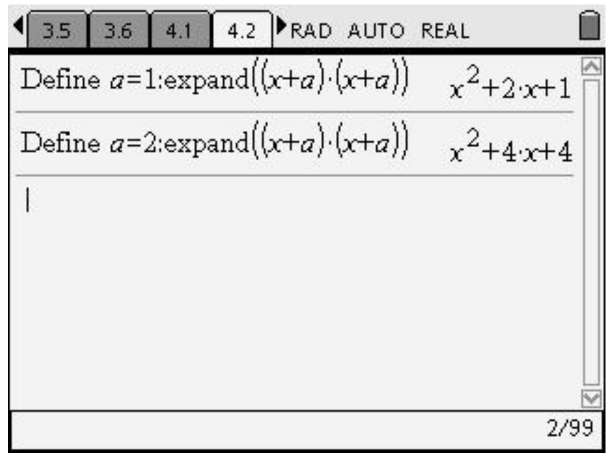


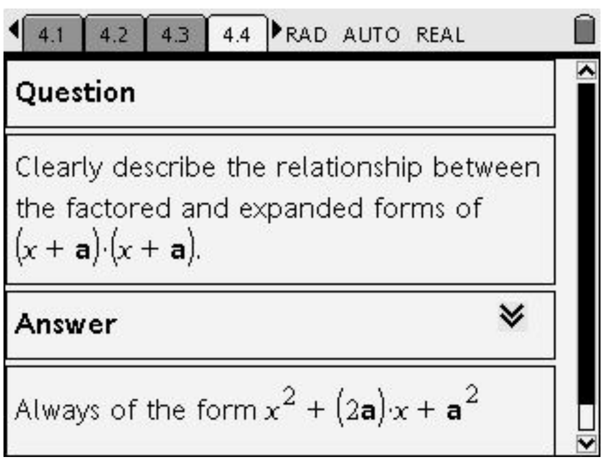


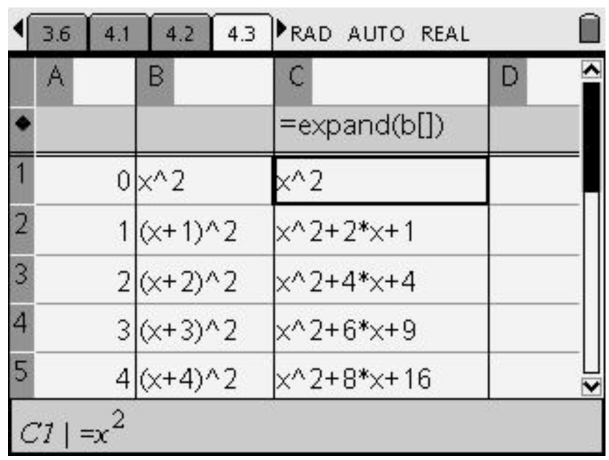


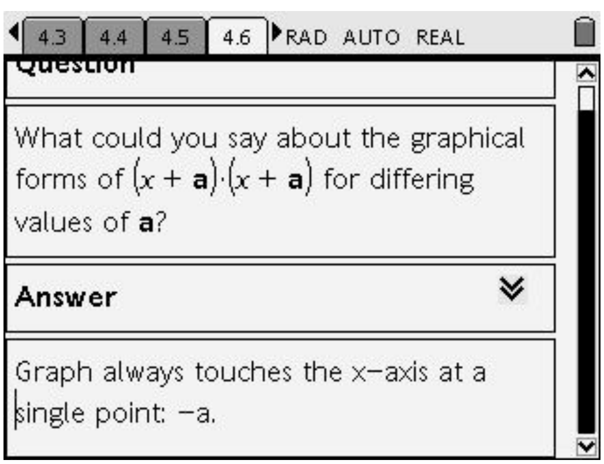
3. Finally, students begin the investigation of **perfect squares**.

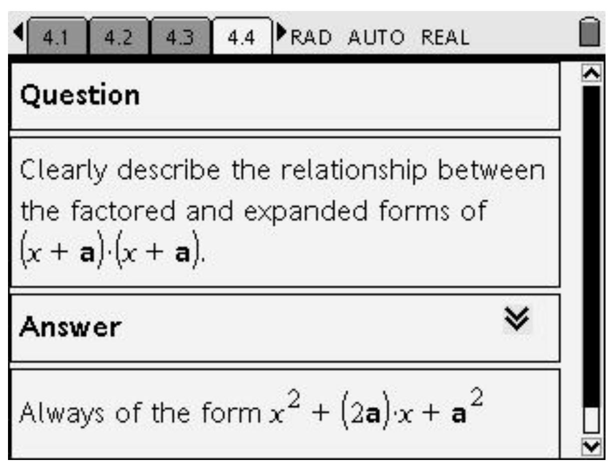
In this exploration of perfect squares, the persistent student belief that $(x+1)^2$ is equivalent to x^2+1 will again be challenged using CAS. CAS suggests equivalence of $(x+1)^2$ and x^2+2x+1 will cause these students to pause and reflect. With each successive perfect square, students will have an opportunity to test their hypotheses around how these equivalent expressions are connected.











Assessment and evaluation

- Answers to student questions that are included in the Student TI-Nspire CAS document
- Observation of student thinking as they work through the activity

Activity extensions

- Students should continue their investigation of more general functions using the tools provided.